Sjournals
Scientific Journals
Scientific Journal of Review (2012) 1(4) 130-146
ISSN 2322-2433

Contents lists available at Sjournals
Scientific Journal of REVIEW
Journal homepage: www.Sjournals.com

Review article

Estimation of the parameters of rayleigh distribution based on record values for censored data

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ARTICLE INFO

Article history:
Received 28 September 2012
Accepted 18 October 2012
Available online 30 October 2012

Keywords:
Record values
Type II censoring
Rayleigh distribution

ABSTRACT

In this paper, we discuss the estimation of location and scale parameters of the Rayleigh distribution based on the Upper record values by the best linear unbiased estimator (BLUE) and an alternative linear estimate, proposed by Gupta (1952). Type II singly and doubly censored samples are used for estimation. Tables of coefficients for best linear estimator and alternative estimator of μ and λ are presented for various choices of censoring for $n \le 8$. Variances and covarinces of estimator of μ and λ for BLUE and for alternative estimator are also presented. The computational formula and procedure used are explained. The purpose of the paper is to compare the performance of the estimation procedures in terms of relative efficiencies. The findings of the study suggest that the BLUE method can be replaced by Gupta method especially in the large samples.

1. Introduction

In our daily life records are always popular in geography and the various natural or social phenomena like sports, weather and business. This importance of the records motivates the necessity to construct mathematical model of records and to develop the corresponding mathematical theory. Awad and Raqab (2000) considered the prediction problem of the future nth records values based on the first m (m < n) observed record values from one-parameter exponential distribution. Four procedures for obtaining prediction intervals for the nth record value were introduced. Lee (2001) discussed the characterization of the exponential distribution under conditional expectations of the record values. Raqab (2002) obtained the exact expressions for single and product moments of record statistics for three parameter generalized exponential distribution. Recurrence relations for single and product moments of record statistics were also established. The variances of the estimators were also presented. The predictors of the future record statistics were also discussed. Ahsanullah, M (2004) has discussed the estimation of the location and scale parameters based upon upper record values of Rayleigh distribution for complete sample without giving the tables of coefficients of the estimates and their variance covarinces. Ahmadi and Doostparast (2008) presented the Bayesian estimation as well as prediction based on k-records when the underlying distribution is assumed to have a general form. Baklizi (2008) proposed the likelihood and Bayesian estimation of the stress-strength reliability based on lower record values from the generalized exponential distribution. The estimators were derived and their properties were studied. Confidence intervals, exact and approximate, as well as the Bayesian credible sets for the stress-strength reliability were obtained. A simulation study was conducted to investigate and compare the performance of the intervals. Shawky (2008) derived the exact form of the probability density function (pdf) and moments of single, double, triple and quadruple of lower record values from Exponentiated Pareto distribution (EPD). Several recurrence relations between single, double, triple and quadruple moments of lower record values from EPD were developed. Sultan (2008) used the lower record values from the inverse Weibull distribution (IWD) to derive and discuss different methods of estimation. Sultan (2008) estimated the parameter of inverse weibull distribution based on record values using different methods of estimation. Ahmadi et al (2009) studied the problem of predicting future *k*-records based on *k*-record data for a large class of distributions, which includes several well-known distributions such as: Exponential, Weibull (one parameter), Pareto, Burr type XII, among others. Akhter and Hirai (2009) estimated the scale parameter of Rayleigh distribution under singly and doubly censored samples using different estimation procedures. Soliman et al (2010) addressed the Bayesian and non-Bayesian estimation problem of the unknown parameter for the inverse Rayleigh distribution based on lower record values. Sultan (2010) considered the estimation and prediction from the inverse Rayleigh distribution on the basis of lower record values.

2. Upper Record values from rayleigh distribution

A record is an entry that is smaller or greater than all previous entries. If an entry that is greater than all previous entries then we call it upper record value. Suppose $x_1, x_2, x_3, \ldots, x_n$ be a sequence of independent and identical distributed random variable from the Rayleigh distribution whose density function is:

$$f(x) = \frac{2x}{\lambda^2}\text{Exp}[-\frac{x^2}{\lambda^2}] \qquad 0<x<\infty$$

Then $y_n = \max\{x_1, x_2, x_3 \ldots x_n\}$ for $n \geq 1$ we call x_j is the upper record value if $y_j > y_{j-1}$,j>1

Let us denote the $E(y_{U(n)}) = \alpha_n$, $\text{Cov}(y_{U(n)}, y_{U(m)}) = \omega_{n.m} - \alpha_n\alpha_m$ *and* $\text{Var}(y_{U(n)}) = \omega_{nn} - (\alpha_n)^2$

The results have has been tabulated in following tables for sample size n up to order 8.

Table 1

Means of the Upper record values from Rayleigh distribution for 1 ≤ n ≤ 8.

N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Means	0.886227	1.329340	1.66168	1.93862	2.18095	2.39904	2.59896	2.78460

Table 2
Variance and Covariance of Upper record values from Rayleigh distribution.

s/r	1	2	3	4	5	6	7	8
1	0.214602							
2	0.155236	0.232854						
3	0.127378	0.191068	0.238834					
4	0.110513	0.165769	0.207212	0.241747				
5	0.0989302	0.148395	0.185494	0.216410	0.243461			
6	0.0903528	0.135529	0.169412	0.197647	0.222353	0.244508		
7	0.0836742	0.125511	0.156889	0.183037	0.205917	0.226509	0.245384	
8	0.0782846	0.117427	0.146784	0.171247	0.192653	0.211919	0.229579	0.245977

Further let $y_{U(r+1)}, y_{U(r+2)}, y_{U(r+3)}, \ldots.. y_{U(n-r_1-r_2)}$ is the sequence of upper record values from Rayleigh distribution with μ and λ as the location and scale parameters respectively, whose density function is:

$$f(x) \quad = \quad \frac{2(x-\mu)}{\lambda^2} \mathrm{Exp}[-\frac{(x-\mu)^2}{\lambda^2}] \qquad \mu \leq x \leq \infty,\ \lambda>0$$

are available for Type II censoring. In which r_1 and r_2 are fixed in advance. r_1 is the missing observation on left and r_2 is the missing observations on right. $(n-r_1-r_2)$ are measured observation. So r_1 is the smallest and r_2 is the largest observations were not observed due to experimental restrictions. Under such condition we have to estimates these parameters by Lloyd's and Gupta method.

3. Estimation of μ and λ by BLUE

Suppose $y_{U(r+1)}, y_{U(r+2)}, y_{U(r+3)}, \ldots.. y_{U(n-r_1-r_2)}$ is the sequence of upper record values from Rayleigh distribution for TypeII censoring are available. We have the best linear unbiased estimators $\overset{*}{\mu}$ and $\overset{*}{\lambda}$ expressed as the linear function of upper record statistics, namely,

$$\overset{*}{\mu} = \sum_{i=1}^{n-r_1-r_2} a_i y_{U(n)} \quad \text{and} \quad \overset{*}{\lambda} = \sum_{i=1}^{n-r_1-r_2} b_i y_{U(n)} \quad \text{where,}$$

$$\mathbf{a} = \frac{\mathbf{1}}{\boldsymbol{\Delta}}\left[\boldsymbol{\alpha}' \, (\mathbf{V}^{(n)})^{-1} \boldsymbol{\alpha} \mathbf{1}' - \left(\mathbf{1}'(\mathbf{V}^{(n)})^{-1}\boldsymbol{\alpha}\right)\boldsymbol{\alpha}'\right](\mathbf{V}^{(n)})^{-1}$$

$$\mathbf{b} = \frac{\mathbf{1}}{\boldsymbol{\Delta}}\left[-\mathbf{1}'(\mathbf{V}^{(n)})^{-1}\boldsymbol{\alpha}\mathbf{1}' + \left(\mathbf{1}'(\mathbf{V}^{(n)})^{-1}\mathbf{1}\boldsymbol{\alpha}\right)\right](\mathbf{V}^{(n)})^{-1}$$

The variance and covariance of the above estimators are

$$\mathrm{Var}(\overset{*}{\mu}) = \frac{\boldsymbol{\lambda}^2 \boldsymbol{\alpha}' \, (\mathbf{V}^{(n)})^{-1} \boldsymbol{\alpha}}{\boldsymbol{\Delta}}$$

$$\mathrm{Var}(\overset{*}{\lambda}) = \frac{\boldsymbol{\lambda}^2 \mathbf{1}'(\mathbf{V}^{(n)})^{-1}\mathbf{1}}{\boldsymbol{\Delta}}$$

$$\mathrm{Cov}(\overset{*}{\mu}, \overset{*}{\lambda}) = -\frac{\boldsymbol{\lambda}^2 \mathbf{1}'(\mathbf{V}^{(n)})^{-1}\boldsymbol{\alpha}}{\boldsymbol{\Delta}}$$

Where $\boldsymbol{\Delta} = [(\boldsymbol{\alpha}' (\mathbf{V}^{(n)})^{-1} \boldsymbol{\alpha})(\mathbf{1}'(\mathbf{V}^{(n)})^{-1}\mathbf{1}) - (\mathbf{1}'(\mathbf{V}^{(n)})^{-1}\boldsymbol{\alpha})^2]$

where $\mathrm{y}^{(n)}$ be the matrix of y_{U}

$$Y = \begin{bmatrix} y_{U(r_1+1)} \\ y_{U(r_1+2)} \\ . \\ . \\ y_{U(n-r_1-r_2+1)} \end{bmatrix} \qquad , \ \alpha' = \begin{bmatrix} \alpha_{U(r_1+1)} & \alpha_{U(r_1+2)} . & . & . & \alpha_{U(n-r_1-r_2+1)} \end{bmatrix}$$

$$V = \begin{bmatrix} v_{r_1+1,r_1+1} & v_{r_1+1,r_1+2} & . & . & . & . & v_{r_1+1,n-r_1-r_2} \\ v_{r_1+1,r_1+2} & v_{r_1+2,r_1+2} & . & . & . & . & v_{r_1+1,n-r_1-r_2} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ v_{n-r_1-r_2+1,r_1+1} & . & . & . & . & . & v_{n-r_1-r_2+1,n-r_1-r_2+1} \end{bmatrix}$$

By using the table 1 and table 2, we computed the coefficients for BLUEs of μ and λ and the variances and covarinces of these estimators for sample size n up to order 8 and presented in the following tables.

Table 3

Showing the coefficients for BLUE of the location parameter based on Upper record values of Rayleigh distribution from singly and doubly censored samples.

n	**r_1**	**r_2**	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
3	0	1	3.0000	-2.0000						
	1	0		4.9999	-3.9999					
4	0	1	2.0000	0.3334	-1.3333					
	0	2	3.0000	-2.0000						
	1	0		3.0000	0.3999	-2.4000				
	1	1		4.9999	-3.9999					
	2	0			7.0001	-6.0001				
5	0	1	1.6364	0.2727	0.1818	-1.0909				
	0	2	2.0000	0.3334	-1.3333					
	0	3	3.0000	-2.0000						
	1	0		2.3077	0.3076	0.2309	-1.8462			
	1	1		3.0000	0.3999	-2.4000				
	1	2		4.9999	-3.9999					
	2	0			4.0000	0.4286	-3.4286			
	2	1			7.0001	-6.0001				
	3	0				8.9999	-7.9999			
6	0	1	1.4411	0.2402	0.1601	0.1201	-0.9607			
	0	2	1.6364	0.2727	0.1818	-1.0909				
	0	3	2.0000	0.3334	-1.3333					
	0	4	3.0000	-2.0000						
	1	0		1.9476	0.2596	0.1948	0.1585	-1.5605		

	1	1		2.3077	0.3076	0.2309	-1.8462			
	1	2		3.0000	0.3999	-2.4000				
	1	3		4.9999	-3.9999					
	2	0			2.9775	0.3190	0.2597	-2.5563		
	2	1			4.0000	0.4286	-3.4286			
	2	2			7.0001	-6.0001				
	3	0				4.9965	0.4519	-4.4984		
	3	1				-8.9992	-7.9999			
	4	0					11.0002	-10.0002		
	0	1	1.3137	0.2189	0.1460	0.1095	0.0891	-0.8772		
	0	2	1.4411	0.2402	0.1601	0.1201	-0.9607			
	0	3	1.6364	0.2727	0.1818	-1.0909				
	0	4	2.0000	0.3334	-1.3333					
	0	5	3.0000	-2.0000						
	1	0		1.7242	0.2298	0.1725	0.1377	0.1154	-1.3796	
	1	1		1.9476	0.2596	0.1948	0.1585	-1.5605		
	1	2		2.3077	0.3076	0.2309	-1.8462			
	1	3		3.0000	0.3999	-2.4000				
7	1	4		4.9999	-3.9999					
	2	0			2.4562	0.2631	0.2102	0.1761	-2.1057	
	2	1			2.9775	0.3190	0.2597	-2.5563		
	2	2			4.0000	0.4286	-3.4286			
	2	3			7.0001	-6.0001				
	3	0				3.6487	0.3237	0.2714	-3.2438	
	3	1				4.9965	0.4519	-4.4484		
	3	2				8.9999	-7.9999			
	4	0					5.9999	0.4564	-5.4556	
	4	1					11.0002	-10.0002		
	5	0						13.0000	-12.0000	
	0	1	1.2245	0.2041	0.1360	0.1020	0.0815	0.0683	-0.8165	
	0	2	1.3137	0.2189	0.1460	0.1095	0.0891	-0.8772		
	0	3	1.4411	0.2402	0.1601	0.1201	-0.9607			
	0	4	1.6364	0.2727	0.1818	-1.0909				
	0	5	2.0000	0.3334	-1.3333					
	0	6	3.0000	-2.0000						
	1	0		1.5695	0.2092	0.1570	0.1253	0.1051	0.0895	-1.2557
	1	1		1.7242	0.2298	0.1725	0.1377	0.1154	-1.3796	
	1	2		1.9476	0.2596	0.1948	0.1585	-1.5605		
	1	3		2.3077	0.3076	0.2309	-1.8462			
	1	4		3.0000	0.3999	-2.4000				
8				4.9999	-3.9999					
	2	0			2.1351	0.2287	0.1827	0.1531	0.1305	-1.8801
	2	1			2.4562	0.2631	0.2102	0.1761	-2.1057	
	2	2			2.9775	0.3190	0.2597	-2.5563		
	2	3			4.0000	0.4286	-3.4286			
	2	4			7.0001	-6.0001				
	3	0				2.9624	0.2628	0.2203	0.1877	-2.6333
	3	1				3.6487	0.3237	0.2714	-3.2438	
	3	2				4.9965	0.4519	-4.4484		
	3	3				8.9992	-7.9999			
	4	0					4.3171	0.3284	0.2798	-3.9253

4	1	5.9999	0.4564	-5.4556	
4	2	11.0002	-10.0002		
5	0		7.0072	0.4465	-6.4537
5	1		13.0000	-12.0000	
6	0			15.0000	-14.0000

Table 4
Showing the coefficients for BLUE of scale parameter Based on Upper record value of Rayleigh distribution from singly and doubly censored samples.

n	r_1	r_2	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
3	0	1	-2.2567	2.2567						
	1	0		-3.0089	3.0089					
4	0	1	-1.2036	-0.2006	1.4042					
	0	2	-2.2567	2.2567						
	1	0		-1.5475	-0.2063	1.7538				
	1	1		-3.0089	3.0089					
	2	0			-3.6109	3.6109				
5	0	1	-0.8441	-0.1407	-0.0938	1.0786				
	0	2	-1.2358	-0.2006	1.4042					
	0	3	-2.2567	2.2567						
	1	0		-1.0581	-0.1410	-0.1059	1.3050			
	1	1		-1.5475	-0.2063	1.7538				
	1	2		-3.0089	3.0089					
	2	0			-1.8345	-0.1965	2.0306			
	2	1			-3.6109	3.6109				
	3	0				-4.1266	4.1266			
6	0	1	-0.6608	-0.1101	-0.0734	-0.0551	0.8994			
	0	2	-0.8441	-0.1407	-0.0938	1.0786				
	0	3	-1.2358	-0.2006	1.4042					
	0	4	-2.2567	2.2567						
	1	0		-0.8117	-0.1082	-0.0812	-0.0668	1.0679		
	1	1		-1.0581	-0.1410	-0.1059	1.3050			
	1	2		-1.5475	-0.2063	1.7538				
	1	3		-3.0089	3.0089					
	2	0			-1.2409	-0.1330	-0.1090	1.4829		
	2	1			-1.8345	-0.1965	2.0306			
	2	2			-3.6109	3.6109				
	3	0				-2.0824	-0.1891	2.2714		
	3	1				-4.1266	4.1266			
	4	0					-4.5853	4.5853		
7	0	1	-0.5475	-0.0912	-0.0608	-0.0456	-0.0379	0.7831		
	0	2	-0.6608	-0.1101	-0.0734	-0.0551	0.8994			
	0	3	-0.8441	-0.1407	-0.0938	1.0786				
	0	4	-1.2358	-0.2006	1.4042					
	0	5	-2.2567	2.2567						
	1	0		-0.6634	-0.0884	-0.0664	-0.0530	-0.0443	0.9156	
	1	1		-0.6608	-0.1101	-0.0734	-0.0551	0.8994		
	1	2		-0.8441	-0.1407	-0.0938	1.0786			
	1	3		-1.2358	-0.2006	1.4042				
	1	4		-2.2567	2.2567					
	2	0			-0.9451	-0.1013	-0.0809	-0.0678	1.1950	
	2	1			-1.2409	-0.1330	-0.1090	1.4829		
	2	2			-1.8345	-0.1965	2.0306			
	2	3			-3.6109	3.6109				

	3	0				-0.6219	-0.0552	-0.0463	2.3541	
	3	1				-2.0824	-0.1891	2.2714		
	3	2				-4.1266	4.1266			
	4	0					-2.3083	-0.1756	2.4839	
	4	1					-4.5853	4.5853		
	5	0						-5.0020	5.0020	
8	0	1	1.2245	0.2041	0.1360	0.1020	0.0815	0.0683	-0.8165	
	0	2	-0.5475	-0.0912	-0.0608	-0.0456	-0.0379	0.7831		
	0	3	-0.6608	-0.1101	-0.0734	-0.0551	0.8994			
	0	4	-0.8441	-0.1407	-0.0938	1.0786				
	0	5	-1.2358	-0.2006	1.4042					
	0	6	-2.2567	2.2567						
	1	0		-0.5636	-0.0751	-0.0564	-0.0450	0.0377	-0.0322	0.8101
	1	1		-0.6634	-0.0884	-0.0664	-0.0530	-0.0443	0.9156	
	1	2		-0.6608	-0.1101	-0.0734	-0.0551	0.8994		
	1	3		-0.8441	-0.1407	-0.0938	1.0786			
	1	4		-1.2358	-0.2006	1.4042				
	1	5		-2.2567	2.2567					
	2	0			-0.7668	-0.0821	-0.0656	-0.0550	-0.0469	1.0164
	2	1			-0.9451	-0.1013	-0.0809	-0.0678	1.1950	
	2	2			-1.2409	-0.1330	-0.1090	1.4829		
	2	3			-1.8345	-0.1965	2.0306			
	2	4			-3.6109	3.6109				
	3	0				-1.0639	-0.0944	-0.0791	-0.0674	1.3048
	3	1				-0.6219	-0.0552	-0.0463	2.3541	
	3	2				-2.0824	-0.1891	2.2714		
	3	3				-4.1266	4.1266			
	4	0					-1.5504	-0.1179	-0.1005	1.7688
	4	1					-2.3083	-0.1756	2.4839	
	4	2					-4.5853	4.5853		
	5	0						-2.5164	-0.1604	2.6768
	5	1						-5.0020	5.0020	
	6	0							-5.3868	5.3668

Table 5

Variances and covariances of BLUEs for location and scale based on the Upper record values from the Rayleigh distribution from singly and doubly censored samples. Each value may be multiplied by λ^2.

N	r_1	r_2	$Var(\mu^*)$	$Var(\lambda^*)$	$Cov(\mu^*, \lambda^*)$
3	0	1	0.999996	0.697649	-0.752250
	1	0	1.999911	0.810787	- 1.203543
4	0	1	0.666661	0.327936	-0.401196
	0	2	0.999996	0.697649	-0.752250
	1	0	1.2000062	0.383624	-0.619001
	1	1	1.999911	0.810787	-1.203543
	2	0	3.000073	0.862590	-1.547532
5	0	1	0.545456	0.209460	-0.281363
	0	2	0.666666	0.327936	-0.401196
	0	3	0.999996	0.697649	-0.752250
	1	0	0.923081	0.242525	-0.423247
	1	1	1.200006	0.383624	-0.619001
	1	2	1.999911	0.810787	-1.203543
	2	0	1.714299	0.411594	-0.786034
	2	1	3.00073	0.862590	-1.547532

	3	0	3.999810	0.892108	-1.834014
6	0	1	0.480354	0.152210	-0.220250
	0	2	0.545456	0.209460	-0.281363
	0	3	0.666661	0.327936	-0.401196
	0	4	0.999996	0.697649	-0.752250
	1	0	0.779031	0.177797	-0.324674
	1	1	0.923081	0.242525	-0.423247
	1	2	1.200006	0.383624	-0.619001
	1	3	1.999911	0.810787	-1.203543
	2	0	1.276087	0.264133	-0.531831
	2	1	1.714299	0.411594	-0.786034
	2	2	3.00073	0.862590	-1.547532
	3	0	2.220653	0.428198	-0.925494
	3	1	3.999810	0.892108	-1.834014
	4	0	4.992129	0.909589	-2.080553
7	0	1	0.437895	0.118543	-0.182500
	0	2	0.480354	0.152210	-0.220250
	0	3	0.545456	0.209460	-0.281363
	0	4	0.666661	0.327936	-0.401196
	0	5	0.999996	0.697649	-0.752250
	1	0	0.689657	0.138431	-0.265359
	1	1	0.779031	0.177797	-0.324674
	1	2	0.923081	0.242525	-0.423247
	1	3	1.200006	0.383624	-0.619001
	1	4	1.999911	0.810787	-1.203543
	2	0	1.052641	0.192170	-0.405025
	2	1	1.276087	0.264133	-0.531831
	2	2	1.714299	0.411594	-0.786034
	2	3	3.00073	0.862590	-1.547532
	3	0	1.621629	0.276407	-0.623953
	3	1	2.220653	0.428198	-0.925494
	3	2	3.999810	0.892108	-1.834014
	4	0	2.727298	0.440098	-1.049381
	4	1	4.992129	0.909589	-2.080553
	5	0	5.986340	0.922588	-2.030336
8	0	1	0.408165	0.096756	-0.157049
	0	2	0.437895	0.118543	-0.182500
	0	3	0.480354	0.152210	-0.220250
	0	4	0.545456	0.209460	-0.281363
	0	5	0.666661	0.327936	-0.401196
	0	6	0.999996	0.697649	-0.752250
	1	0	0.627803	0.112688	-0.225455
	1	1	0.689657	0.138431	-0.265359
	1	2	0.779031	0.177797	-0.324674
	1	3	0.923081	0.242525	-0.423247
	1	4	1.200006	0.383624	-0.619001
	1	5	1.999911	0.810787	-1.203543
	2	0	0.915036	0.149731	-0.328606
	2	1	1.052641	0.192170	-0.405025
	2	2	1.276087	0.264133	-0.531831
	2	3	1.714299	0.411594	-0.786034

2	4	3.00073	0.862590	-1.547532
3	0	1.316622	0.201522	-0.472823
3	1	1.621629	0.276407	-0.623953
3	2	2.220653	0.428198	-0.925494
3	3	3.999810	0.892108	-1.834014
4	0	1.962644	0.284837	-0.704822
4	1	2.727298	0.440098	-1.049381
4	2	4.992129	0.909589	-2.080553
5	0	3.226817	0.447871	-1.158808
5	1	5.986340	0.922588	-2.030336
6	0	6.999712	0.934444	-2.513720

4. An alternative linear estimate proposed by Gupta (1952)

Lloyd's procedure requires full knowledge of the expectations and the variance covariance matrix of the record values. The covariance be specially may be different to determine. Gupta (1952) has proposed a very simple method applicable when only the expectations are known. The coefficients of these linear estimates are obtained by assuming the variance matrix to be a unit matrix. Let the linear estimates be

$$\overset{*}{\mu} = \sum_{i=r_1+1}^{n-r_2} b_i x_{(i)}^n \quad \text{and} \quad \overset{*}{\lambda} = \sum_{i=r_1+1}^{n-r_2} c_i x_{(i)}^n$$

where

$$b_i = \frac{1}{n-r_1-r_2} - \frac{\overline{\mu_k}(\mu_i - \overline{\mu_k})}{\sum_{i=r_1+1}^{n-r_2} (\mu_i - \overline{\mu_k})^2}, \quad c_i = \frac{(\mu_i - \overline{\mu_k})}{\sum_{i=r_1+1}^{n-r_2} (\mu_i - \overline{\mu_k})^2} \quad \text{and} \quad \overline{\mu_k} = \frac{1}{n-r_1-r_2} \sum_{j=r_1+1}^{n-r_2} \mu_j$$

In matrix notation the variance of the estimates can also be written as

$$\mathbf{V}(\overset{*}{\boldsymbol{\mu}}) = \boldsymbol{\lambda}^2\, \mathbf{b}'\mathbf{V}\mathbf{b}$$

$$\mathbf{V}(\overset{*}{\boldsymbol{\lambda}}) = \boldsymbol{\lambda}^2\, \mathbf{c}'\mathbf{V}\mathbf{c}$$

By making use of table I and II, we determine the coefficients of the estimates and variances of the estimates are presented in table VI, VII and VIII respectively.

Table 6

Showing the coefficients for alternative linear estimate of location parameters based on Upper Records values of Rayleigh distribution from singly and doubly censored samples

n	r_1	r_2	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
3	0	1	3.0000	2.0000						
	1	0		5.0000	-3.9999					
4	0	1	2.0676	0.1757	-1.2432					
	0	2	3.0000	-2.0000						
	1	0		3.1044	0.1703	-2.2747				
	1	1		4.9999	-3.9999					
	2	0			7.0002	-6.0002				
5	0	1	1.5903	0.5442	-0.2404	-0.8941				
	0	2	2.0676	0.1757	-1.2432					
	0	3	3.0000	-2.0000						
	1	0		2.2275	0.7616	-2.2747				

	1	1		3.1044	0.1703	-2.2747				
	1	2		4.9999	-3.9999					
	2	0			4.1213	0.1687	-3.2900			
	2	1			7.0001	-6.0001				
	3	0				8.9999	-7.9999			
6	0	1	1.2981	0.6158	0.1040	-0.3224	-0.6955			
	0	2	1.5903	0.5442	-0.2404	-0.8941				
	0	3	2.0676	0.1757	-1.2432					
	0	4	3.0000	-2.0000						
	1	0		1.7297	0.8418	0.1020	-0.5455	-1.1281		
	1	1		2.2275	0.7616	-2.2747				
	1	2		3.1044	0.1703	-2.2747				
	1	3		4.9999	-3.9999					
	2	0			2.8456	0.9707	-0.6699	-2.1464		
	2	1			4.1213	0.1687	-3.2900			
	2	2			7.0001	-6.0001				
	3	0				5.1310	0.1678	-4.2989		
	3	1				8.9999	-7.9999			
	4	0					11.0002	-10.0002		
7	0	1	1.0998	0.6113	0.2449	-0.0604	-0.3276	-0.5680		
	0	2	1.2981	0.6158	0.1040	-0.3224	-0.6955			
	0	3	1.5903	0.5442	-0.2404	-0.8941				
	0	4	2.0676	0.1757	-1.2432					
	0	5	3.0000	-2.0000						
	1	0		1.4113	0.8107	0.3103	-0.1276	-0.5217	-0.8830	
	1	1		1.7297	0.8418	0.1020	-0.5455	-1.1281		
	1	2		2.2275	0.7616	-2.2747				
	1	3		3.1044	0.1703	-2.2747				
	1	4		4.9999	-3.9999					
	2	0			2.1458	1.0554	0.1012	-0.7576	-1.5448	
	2	1			2.8456	0.9707	-0.6699	-2.1464		
	2	2			4.1213	0.1687	-3.2900			
	2	3			7.0001	-6.0001				
	3	0				3.4563	1.1762	-0.8758	-2.7568	
	3	1				5.1310	0.1678	-4.2989		
	3	2				8.9999	-7.9999			
	4	0					6.1373	0.1675	-5.3049	
	4	1					11.0002	-10.0002		
	5	0						13.0000	-12.0000	
8	0	1	0.9560	0.5846	0.3061	0.0739	-0.1292	-0.3120	-0.4795	
	0	2	1.0998	0.6113	0.2449	-0.0604	-0.3276	-0.5680		
	0	3	1.2981	0.6158	0.1040	-0.3224	-0.6955			
	0	4	1.5903	0.5442	-0.2404	-0.8941				
	0	5	2.0676	0.1757	-1.2432					
	0	6	3.0000	-2.0000						
	1	0		1.1910	0.7546	0.3910	0.0728	-0.2136	-0.4761	-0.7198
	1	1		1.4113	0.8107	0.3103	-0.1276	-0.5217	-0.8830	
	1	2		1.7297	0.8418	0.1020	-0.5455	-1.1281		
	1	3		2.2275	0.7616	-2.2747				
	1	4		3.1044	0.1703	-2.2747				
	1	5		4.9999	-3.9999					
	2	0			1.7107	0.9968	0.3721	-0.1901	-0.7055	-1.1840
	2	1			2.1458	1.0554	0.1012	-0.7576	-1.5448	
	2	2			2.8456	0.9707	-0.6699	-2.1464		
	2	3			4.1213	0.1687	-3.2900			
	2	4			7.0001	-6.0001				
	3	0				2.5558	1.2636	0.1008	-0.9652	-1.9550

3	1	3.4563	1.1762	-0.8758	-2.7568	
3	2	5.1310	0.1678	-4.2989		
3	3	8.9999	-7.9999			
4	0		4.0635	1.3801	-1.0797	-3.3639
4	1		6.1373	0.1675	-5.3049	
4	2		11.0002	-10.0002		
5	0			7.1417	0.1673	-6.3090
5	1			13.0000	-12.0000	
6	0				15.0000	-14.0000

Tables 7

Showing the coefficients for alternative linear estimate of scale parameter based on Upper record values of Rayleigh distribution from singly and doubly censored samples.

n	r_1	r_2	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
3	0	1	-2.2568	2.2568						
	1	0		-3.0090	3.0090					
4	0	1	-1.3418	0.1220	1.2199					
	0	2	-2.2568	2.2568						
	1	0		-1.6864	0.0992	1.5872				
	1	1		-3.0090	3.0090					
	2	0			-3.6109	3.6109				
5	0	1	-0.9218	-0.2024	0.3373	0.7869				
	0	2	-1.3418	0.1220	1.2199					
	0	3	-2.2568	2.2568						
	1	0		-1.1124	-0.2878	0.3994	1.0008			
	1	1		-1.6864	0.0992	1.5872				
	1	2		-3.0090	3.0090					
	2	0			-1.9657	0.0854	1.8802			
	2	1			-3.6109	3.6109				
	3	0				-4.1266	4.1266			
6	0	1	-0.6866	-0.2600	0.0600	0.3266	0..559919			
	0	2	-0.9218	-0.2024	0.3373	0.7869				
	0	3	-1.3418	0.1220	1.2199					
	0	4	-2.2568	2.2568						
	1	0		-0.8043	-0.3375	0.0515	0.3919	0.6983		
	1	1		-1.1124	-0.2878	0.3994	1.0008			
	1	2		-1.6864	0.0992	1.5872				
	1	3		-3.0090	3.0090					
	2	0			-1.2692	-0.3524	0.4498	1.1718		
	2	1			-1.9657	0.0854	1.8802			
	2	2			-3.6109	3.6109				
	3	0				-2.2080	0.0762	2.1318		
	3	1				-4.1266	4.1266			
	4	0					-4.5853	4.5853		
7	0	1	-0.5386	-0.2566	-0.0452	0.1311	0.2853	0.4240		
	0	2	-0.6866	-0.2600	0.0600	0.3266	0..559919			
	0	3	-0.9218	-0.2024	0.3373	0.7869				

	0	4	-1.3418	0.1220	1.2199					
	0	5	-2.2568	2.2568						
	1	0		-0.6167	-0.3191	-0.0712	0.1458	0.3411	0.5201	
	1	1		-0.8043	-0.3375	0.0515	0.3919	0.6983		
	1	2		-1.1124	-0.2878	0.3994	1.0008			
	1	3		-1.6864	0.0992	1.5872				
	1	4		-3.0090	3.0090					
	2	0			-0.9026	-0.3968	0.0458	0.4442	0.8093	
	2	1			-1.2692	-0.3524	0.4498	1.1718		
	2	2			-1.9657	0.0854	1.8802			
	2	3			-3.6109	3.6109				
	3	0				-1.4067	-0.4064	0.4939	1.3191	
	3	1				-2.2080	0.0762	2.1318		
	3	2				-4.1266	4.1266			
	4	0					-2.4254	0.0693	2.3561	
	4	1					-4.5853	4.5853		
	5	0						-5.0020	5.0020	
	0	1	-0.4380	-0.2380	-0.0879	0.0371	0.1465	0.2450	0.3353	
	0	2	-0.5386	-0.2566	-0.0452	0.1311	0.2853	0.4240		
	0	3	-0.6866	-0.2600	0.0600	0.3266	0..559919			
	0	4	-0.9218	-0.2024	0.3373	0.7869				
	0	5	-1.3418	0.1220	1.2199					
	0	6	-2.2568	2.2568						
	1	0		-0.4926	-0.2875	-0.1166	0.0329	0.1675	0.2909	0.4055
	1	1		-0.6167	-0.3191	-0.0712	0.1458	0.3411	0.5201	
	1	2		-0.8043	-0.3375	0.0515	0.3919	0.6983		
	1	3		-1.1124	-0.2878	0.3994	1.0008			
	1	4		-1.6864	0.0992	1.5872				
	1	5		-3.0090	3.0090					
	2	0			-0.6830	-0.3672	-0.0909	0.1578	0.3858	0.5975
8	2	1			-0.9026	-0.3968	0.0458	0.4442	0.8093	
	2	2			-1.2692	-0.3524	0.4498	1.1718		
	2	3			-1.9657	0.0854	1.8802			
	2	4			-3.6109	3.6109				
	3	0				-0.9896	-0.4468	0.0417	0.4895	0.9053
	3	1				-1.4067	-0.4064	0.4939	1.3191	
	3	2				-2.2080	0.0762	2.1318		
	3	3				-4.1266	4.1266			
	4	0					-1.5310	-0.4537	0.5338	1.4508
	4	1					-2.4254	0.0693	2.3561	
	4	2					-4.5853	4.5853		
	5	0						-2.6245	0.0640	2.5604
	5	1						-5.0020	5.0020	
	6	0							-5.3868	5.3868

Tables 8
Variances of alternative linear estimate for location and scale based on the Upper record values from the Rayleigh distribution from singly and doubly censored samples, each value may be multiplied by λ^2.

n	r_1	r_2	$Var(\mu^*)$	$Var(\lambda^*)$
3	0	1	0.999969	0.697655
	1	0	1.999928	0.810793
4	0	1	0.668187	0.334310
	0	2	0.999969	0.697655
	1	0	1.202187	0.387482
	1	1	1.999928	0.810793
	2	0	3.000080	0.862590
5	0	1	0.553555	0.218249
	0	2	0.668187	0.334310
	0	3	0.999969	0.697655
	1	0	0.939199	0.253486
	1	1	1.202187	0.387482
	1	2	1.999928	0.810793
	2	0	1.716405	0.414069
	2	1	3.000080	0.862590
	3	0	3.999897	0.892108
6	0	1	0.494683	0.161591
	0	2	0.553555	0.218249
	0	3	0.668187	0.334310
	0	4	0.999969	0.697655
	1	0	0.805608	0.187845
	1	1	0.939199	0.253486
	1	2	1.202187	0.387482
	1	3	1.999928	0.810793
	2	0	1.299863	0.271944
	2	1	1.716405	0.414069
	2	2	3.000080	0.862590
	3	0	2.222661	0.429950
	3	1	3.999897	0.892108
	4	0	4.992129	0.909589
7	0	1	0.458576	0.121811
	0	2	0.494683	0.161591
	0	3	0.553555	0.218249
	0	4	0.668187	0.334310
	0	5	0.999969	0.697655
	1	0	0.723676	0.148928
	1	1	0.805608	0.187845
	1	2	0.939199	0.253486
	1	3	1.202187	0.387482
	1	4	1.999928	0.810793
	2	0	1.090566	0.202214
	2	1	1.299863	0.271944
	2	2	1.716405	0.414069
	2	3	3.000080	0.862590
	3	0	1.652426	0.283819

	3	1	2.222661	0.429950
	3	2	3.999897	0.892108
	4	0	2.729027	0.441340
	4	1	4.992129	0.909589
	5	0	5.986340	0.922588
	0	1	0.434104	0.106055
	0	2	0.458576	0.121811
	0	3	0.494683	0.161591
	0	4	0.553555	0.218249
	0	5	0.668187	0.334310
	0	6	0.999969	0.697655
	1	0	0.667546	0.123142
	1	1	0.723676	0.148928
	1	2	0.805608	0.187845
	1	3	0.939199	0.253486
	1	4	1.202187	0.387482
	1	5	1.999928	0.810793
	2	0	0.962337	0.160588
8	2	1	1.090566	0.202214
	2	2	1.299863	0.271944
	2	3	1.716405	0.414069
	2	4	3.000080	0.862590
	3	0	1.365675	0.211575
	3	1	1.652426	0.283819
	3	2	2.222661	0.429950
	3	3	3.999897	0.892108
	4	0	2.000408	0.292047
	4	1	2.729027	0.441340
	4	2	4.992129	0.909589
	5	0	3.228206	0.448767
	5	1	5.986340	0.922588
	6	0	6.999712	0.934444

4. Relative efficiency

Efficiency is defined as the ratio of variances of the estimator of BLUEs to variances of the estimator to alternate estimates(Gupta) for all choices of censored data for sample size up to order 8. These efficiencies are presented in table that shows the efficiency in all cases is very high we therefore can replace the Gupta method with Lloyd method which can’t be used for large sample and when the exact variance matrix is not known.

Table 9
Shows the efficiency of BLUE to Gupta.

n	R_1	r_2	$\mathrm{Var}(\mu^*)\mathrm{Lloyd}$	$\mathrm{Var}(\mu^*)\mathrm{Gupta}$	Efficiency (%)	$\mathrm{Var}(\lambda^*)\mathrm{Lloyd}$	$\mathrm{Var}(\lambda^*)\mathrm{Gupta}$	Efficiency (%)
3	0	1	0.999996	0.999969	100.003	0.697649	0.697655	99.999
	1	0	1.999911	1.999928	99.999	0.810787	0.810793	99.999
	0	1	0.666661	0.668187	99.772	0.327936	0.334310	98.030
4	0	2	0.999996	0.999969	100.003	0.697649	0.697655	99.999
	1	0	1.2000062	1.202187	99.819	0.383624	0.387482	99.004

	1	1	1.999911	1.999928	99.999	0.810787	0.810793	99.999
	2	0	3.000073	3.000080	100.000	0.862590	0.862590	100.000
5	0	1	0.545456	0.553555	98.537	0.209460	0.218249	95.973
	0	2	0.666666	0.668187	99.772	0.327936	0.334310	98.093
	0	3	0.999996	0.999969	100.003	0.697649	0.697655	99.999
	1	0	0.923081	0.939199	98.284	0.242525	0.253486	95.676
	1	1	1.200006	1.202187	99.819	0.383624	0.387482	99.004
	1	2	1.999911	1.999928	99.999	0.810787	0.810793	99.999
	2	0	1.714299	1.716405	99.877	0.411594	0.414069	99.402
	2	1	3.00073	3.000080	100.000	0.862590	0.862590	100.000
	3	0	3.999810	3.999897	99.998	0.892108	0.892108	100.000
6	0	1	0.480354	0.494683	97.103	0.152210	0.161591	94.195
	0	2	0.545456	0.553555	98.537	0.209460	0.218249	95.973
	0	3	0.666661	0.668187	99.772	0.327936	0.334310	98.093
	0	4	0.999996	0.999969	100.003	0.697649	0.697655	99.999
	1	0	0.779031	0.805608	96.701	0.177797	0.187845	94.651
	1	1	0.923081	0.939199	98.284	0.242525	0.253486	95.676
	1	2	1.200006	1.202187	99.819	0.383624	0.387482	99.004
	1	3	1.999911	1.999928	99.999	0.810787	0.810793	99.999
	2	0	1.276087	1.299863	98.171	0.264133	0.271944	97.128
	2	1	1.714299	1.716405	99.877	0.411594	0.414069	99.402
	2	2	3.00073	3.000080	100.000	0.862590	0.862590	100.000
	3	0	2.220653	2.222661	99.910	0.428198	0.429950	99.593
	3	1	3.999810	3.999897	99.998	0.892108	0.892108	100.000
	4	0	4.992129	4.992129	100.000	0.909589	0.909589	100.000
7	0	1	0.437895	0.458576	95.490	0.118543	0.121811	97.317
	0	2	0.480354	0.494683	97.103	0.152210	0.161591	94.195
	0	3	0.545456	0.553555	98.537	0.209460	0.218249	95.973
	0	4	0.666661	0.668187	99.772	0.327936	0.334310	98.093
	0	5	0.999996	0.999969	100.003	0.697649	0.697655	99.999
	1	0	0.689657	0.723676	95.299	0.138431	0.148928	92.952
	1	1	0.779031	0.805608	96.701	0.177797	0.187845	94.651
	1	2	0.923081	0.939199	98.284	0.242525	0.253486	95.676
	1	3	1.200006	1.202187	99.819	0.383624	0.387482	99.004
	1	4	1.999911	1.999928	99.999	0.810787	0.810793	99.999
	2	0	1.052641	1.090566	96.522	0.192170	0.202214	95.033
	2	1	1.276087	1.299863	98.170	0.264133	0.271944	97.128
	2	2	1.714299	1.716405	99.877	0.411594	0.414069	99.402
	2	3	3.00073	3.000080	100.000	0.862590	0.862590	100.000
	3	0	1.621629	1.652426	98.136	0.276407	0.283819	97.388
	3	1	2.220653	2.222661	99.910	0.428198	0.429950	99.593
	3	2	3.999810	3.999897	100.002	0.892108	0.892108	100.000
	4	0	2.727298	2.729027	99.937	0.440098	0.441340	99.719
	4	1	4.992129	4.992129	100.000	0.909589	0.909589	100.000
	5	0	5.986340	5.986340	100.000	0.922588	0.922588	100.000
8	0	1	0.408165	0.434104	94.025	0.096756	0.106055	91.232
	0	2	0.437895	0.458576	95.490	0.118543	0.121811	97.317
	0	3	0.480354	0.494683	97.103	0.152210	0.161591	94.195
	0	4	0.545456	0.553555	98.537	0.209460	0.218249	95.973
	0	5	0.666661	0.668187	99.772	0.327936	0.334310	98.093

0	6	0.999996	0.999969	100.003	0.697649	0.697655	99.999
1	0	0.627803	0.667546	94.046	0.112688	0.123142	91.511
1	1	0.689657	0.723676	95.299	0.138431	0.148928	92.952
1	2	0.779031	0.805608	96.701	0.177797	0.187845	94.651
1	3	0.923081	0.939199	98.284	0.242525	0.253486	95.676
1	4	1.200006	1.202187	99.819	0.383624	0.387482	99.004
1	5	1.999911	1.999928	99.999	0.810787	0.810793	99.999
2	0	0.915036	0.962337	95.085	0.149731	0.160588	93.239
2	1	1.052641	1.090566	96.522	0.192170	0.202214	95.033
2	2	1.276087	1.299863	98.170	0.264133	0.271944	97.128
2	3	1.714299	1.716405	99.877	0.411594	0.414069	99.402
2	4	3.00073	3.000080	100.000	0.862590	0.862590	100.000
3	0	1.316622	1.365675	96.408	0.201522	0.211575	95.248
3	1	1.621629	1.652426	98.136	0.276407	0.283819	97.388
3	2	2.220653	2.222661	99.910	0.428198	0.429950	99.593
3	3	3.999810	3.999897	100.002	0.892108	0.892108	100.000
4	0	1.962644	2.000408	98.112	0.284837	0.292047	97.531
4	1	2.727298	2.729027	99.937	0.440098	0.441340	99.719
4	2	4.992129	4.992129	100.000	0.909589	0.909589	100.000
5	0	3.226817	3.228206	99.957	0.447871	0.448767	99.800
5	1	5.986340	5.986340	100.000	0.922588	0.922588	100.000
6	0	6.999712	6.999712	100.000	0.934444	0.934444	100.000

Acknowledgement

We are very grateful to professor ABDUL-SAMAD-HIRAI former director and professor of statistics institute of statistics, University of Punjab (new campus) under his kind guidance this work has been completed.

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