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Original article

Objective bayesian analysis for the gompertz distribution under doudly type II cesored data

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ABSTRACT

Trimmed samples are widely utilized in several areas of statistical practice, especially when some sample values at either or both extremes might have been adulterated. In this article, the problem of estimating the parameter of Gompertz distribution based on trimmed samples under informative and non-informative priors has been addressed. The problem discussed using Bayesian approach to estimate the parameter of Gompertz distribution. We have examined Bayes estimates under symmetric and asymmetric loss functions. The explicit expressions for estimator and risk are developed under all loss functions. Elicitation of hyperparameter through prior predictive approach is also discussed. Posterior Predictive distributions and Credible Intervals are also derived under different priors. The influence of parametric value on the estimate and risk is also discussed. Finally, to assess the performance of the estimators, numerical results using Monte Carlo simulation study are reported.

1. Introduction

Gompertz probability distribution has many useful applications in areas of the technology, medical, biological, and natural sciences (especially in failure and survival analysis). This distribution was first introduced by Gompertz (1825).

The distribution function of Gompertz probability distribution is given by

$$F(x)=1-\exp\left[-\lambda\left\{\exp(x)-1\right\}\right],\ \ \lambda>0, x>0. \tag{1}$$

And the corresponding pdf of (1) distribution has the following form:

$$f(x)=\lambda\exp(x)\exp\left[-\lambda\left\{\exp(x)-1\right\}\right],\ \ \lambda>0, x>0. \tag{2}$$

where λ is the scale parameter. Trimmed samples are widely employed in several areas of statistical practice, especially when some sample values at either or both extremes might have been contaminated. The problem of estimating the parameters of power function distribution based on a trimmed sample and prior information has been considered in this paper. There are a few works available in literature on the Bayesian analysis of the Gompertz probability distribution and its mixture. Soliman et al. (2012) studied the Bayes and frequentist estimators for the two-parameter Gompertz distribution (GD), as well as the reliability and hazard rate functions, using progressive first-failure censoring plan. Jaheen (2003) considered the Bayesian analysis of record statistics from the Gompertz model. Gordon (1990) derived maximum likelihood estimation for mixtures of two Gompertz distributions when censoring occurs. Wu, et al. (2003) discussed the point and interval estimations for the Gompertz distribution under progressive type-II censoring. Feroze and Aslam (2012) studied Bayesian analysis of Gumbel type II distribution under doubly censored samples using different loss functions. Sindhu et al. (2013) studied the Bayesian and non-Bayesian estimation for the shape parameter of the Kumaraswamy distribution under type-II censored samples.

The objective of this paper is to obtain the estimators of the unknown parameter of the Gompertz distribution based on doubly censored type II. The rest of paper is organized as follows. In section 2, the posterior distributions have been derived under non-informative and informative priors. Estimation of parameter has been discussed in section 3. Credible intervals have been derived in Section 4. Method of Elicitation of the hyper-parameters via prior predictive approach has been discussed in section 5. Posterior predictive distributions are derived in section 6. Simulation study is conducted in section 7. The conclusions regarding the study have been presented in section 8.

2. Prior and posterior distributions

Some data may not be observed, a known number of observation in an ordered sample are missing at both ends in failure censored experiments, the observations are the smallest r and the largest r_s are random then data collected will be $x_{(r+1)}\le x_{(r+2)}\le\ldots\le x_{(n-s)}$ and the likelihood function in double censored type II takes the following form:

$$L(x,\lambda)=\frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s} f\left(x_{(i)},\lambda\right)\left\{F\left(x_{(r)},\lambda\right)\right\}^{r-1}\left\{1-F\left(x_{(s)},\lambda\right)\right\}^{n-s}.$$

$$L(x,\lambda)=\frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s}\left\{\lambda\exp(x)\exp\left[-\lambda\left\{\exp(x)-1\right\}\right]\right\}$$
$$\times\left\{1-\exp\left[-\lambda\left\{\exp(x)-1\right\}\right]\right\}^{r-1}\left\{\exp\left[-\lambda\left\{\exp(x)-1\right\}\right]\right\}^{n-s},$$

$$L(x,\lambda) \propto \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \lambda^{s-r+1} \exp\left[-\lambda\left\{\sum_{i=r}^{s}\{\exp(x_i)-1\} + k\{\exp(x_r)-1\} + (n-s)\{\exp(x_s)-1\}\right\}\right],$$

$$L(x,\lambda) \propto \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \lambda^{\tau} \exp\left[-\lambda\phi_i\left(x_{(i)}\right)\right], \tag{3}$$

where $\tau = s - r + 1,$

and $\phi_i\left(x_{(i)}\right) = \sum_{i=r}^{s}\{\exp(x_i)-1\} + k\{\exp(x_r)-1\} + (n-s)\{\exp(x_s)-1\}.$

2.1. Posterior distribution under non-informative prior

The uniform and Jeffreys prior are the example of non-informative prior which materializes the use of the Bayesian estimation methods when no prior information is available. The posterior distribution under the assumption of uniform and Jeffreys priors have been derived and presented in the following.

Uniform prior reflects the lack of prior information and the Bayesian methodology can still work. Uniform prior may be proper or improper. Even if Uniform prior is improper, we can still have a proper posterior. Equation (4) presents an improper prior while the posterior given in equation (5) is proper one having total area uner the curve equals to unity. The uniform prior for λ is defined as:

$$p(\lambda) \propto k, \quad \lambda > 0. \tag{4}$$

The posterior distribution under the assumption of uniform prior is:

$$p(\lambda\,|\mathbf{x}) = \frac{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \lambda^{\tau} \exp\left[-\lambda\left\{\phi_i\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}, \quad \lambda > 0. \tag{5}$$

The Jeffreys prior has been derived to be:

$$p(\lambda) \propto \frac{1}{\lambda}, \quad \lambda > 0. \tag{6}$$

The posterior distribution under the assumption of Jeffreys prior is:

$$p(\lambda\,|\mathbf{x}) = \frac{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \lambda^{\tau-1} \exp\left[-\lambda\left\{\phi_i\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \frac{\Gamma(\tau)}{\left\{\phi\left(x_{(i)}\right)\right\}^{\tau}}}, \quad \lambda > 0. \tag{7}$$

2.2. Posterior distribution under informative prior

In case of informative prior, the use of prior information is equivalent to add a number of observations to the given sample size and hence leads to a reduction of posterior risks of the Bayes estimates based on the said informative prior. Bolstad (2004) studied a method to evaluate the worth of prior information in terms of the number of additional observations supposed to be added to the given sample size.

The informative prior for the parameter λ is assumed to be exponential distribution:

$$p(\lambda)=me^{-\lambda m},\ \ \lambda>0. \tag{8}$$

The posterior distribution under the assumption of exponential prior is:

$$p(\lambda\,|\mathbf{x})=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\lambda^{\tau}\exp\left[-\lambda\left\{m+\phi_i\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+1)}{\left\{m+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}},\ \ \lambda>0. \tag{9}$$

The informative prior for the parameter λ is assumed to be gamma distribution:

$$p(\lambda)=\frac{b^a}{\Gamma(a)}\lambda^{a-1}e^{-b\lambda},\ a,b,\lambda>0. \tag{10}$$

The posterior distribution under the assumption of gamma prior is:

$$p(\lambda\,|\mathbf{x})=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\lambda^{\tau+a-1}\exp\left[-\lambda\left\{b+\phi_i\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+a)}{\left\{b+\phi_i\left(x_{(i)}\right)\right\}^{\tau+a}}},\ \ a,b,\lambda>0. \tag{11}$$

The informative prior for the parameter λ is assumed to be Inverse Levy distribution:

$$p(\lambda)=\sqrt{\frac{c}{2\pi}}\lambda^{-\frac{1}{2}}\exp\left[-\left(\frac{c\lambda}{2}\right)\right],\ c>0. \tag{12}$$

The posterior distribution under the assumption of Inverse Levy prior is:

$$p(\lambda\,|\mathbf{x})=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\lambda^{\tau-1/2}\exp\left[-\lambda\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma\left(\tau+\frac{1}{2}\right)}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}^{\tau+\frac{1}{2}}}},\ \ c,\lambda>0. \tag{13}$$

It is obvious that the posterior density function under non-informative and informative prior is recognized as the mixture of gamma density functions.

3. Bayes estimators and posterior risks under different loss functions

From a decision-theoretic view point, in order to select the best estimator, a loss function must be specified and is used to represent a penalty associated with each of the possible estimates. This section enlightens the derivation of the Bayes Estimator (BE) and corresponding Posterior Risks (PR) under different loss functions. The Bayes estimators are evaluated under Squared Error Loss Function (SELF), Precautionary Loss Function (PLF), Weighted Squared Error Loss Function (WSELF), Quasi-Quadratic Loss Function (QQLF), Squared-Log Error Loss Function (SLELF), and Entropy Loss Function (ELF). The Bayes Estimator (BE) and corresponding Posterior Risks (PR) under different loss functions are given in the following Table.

Table 1
Bayes estimator and posterior risks under different loss functions.

Loss Function= $L\left(\lambda,\ \hat{\lambda}\right)$	**Bayes Estimator**	**Posterior Risk**
SELF: $\left(\lambda-\hat{\lambda}\right)^2$	$E\left(\lambda\mid\mathbf{x}\right)$	$Var\left(\lambda\mid\mathbf{x}\right)$
PLF: $\dfrac{\left(\lambda-\hat{\lambda}\right)^2}{\hat{\lambda}}$	$\sqrt{E\left(\lambda^2\mid\mathbf{x}\right)}$	$2\left\{\sqrt{E\left(\lambda^2\mid\mathbf{x}\right)}-E\left(\lambda\mid\mathbf{x}\right)\right\}$
WSELF: $\dfrac{\left(\lambda-\hat{\lambda}\right)^2}{\lambda}$	$\left\{E\left(\lambda^{-1}\mid\mathbf{x}\right)\right\}^{-1}$	$E\left(\lambda\mid\mathbf{x}\right)-\left\{E\left(\lambda^{-1}\mid\mathbf{x}\right)\right\}^{-1}$
QQLF: $\left(e^{-c\hat{\lambda}}-e^{-c\lambda}\right)^2$	$\dfrac{-1}{c}\ln\left\{E\left(e^{-c\lambda}\mid\mathbf{x}\right)\right\}$	$E\left(e^{-c\lambda}\right)-\left\{E\left(e^{-c\lambda}\right)\right\}^2$
SLELF: $\left(\ln\hat{\lambda}-\ln\lambda\right)^2$	$\exp\left\{\mathrm{E}\left(\ln\lambda\mid\mathbf{x}\right)\right\}$	$\mathrm{E}\left\{\left(\ln\lambda\mid\mathbf{x}\right)\right\}^2-\left\{\mathrm{E}\left(\ln\lambda\mid\mathbf{x}\right)\right\}^2$
ELF: $b\left\{\left(\dfrac{\hat{\lambda}}{\lambda}\right)-\ln\left(\dfrac{\hat{\lambda}}{\lambda}\right)-1\right\}$	$\left\{E\left(\lambda^{-1}\mid\mathbf{x}\right)\right\}^{-1}$	$\ln\left\{E\left(\lambda^{-1}\mid\mathbf{x}\right)\right\}+E\left(\ln\lambda\right)$

The Bayes Estimators and Posterior Risks under uniform prior are:

$$\hat{\lambda}_{SELF}=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+2)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}},$$

$$\rho\left(\hat{\lambda}_{SELF}\right)=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+3)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+3}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}-\left[\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+2)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}\right]^2.$$

$$\hat{\lambda}_{PLF}=\sqrt{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+3)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+3}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\dfrac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}},$$

$$\rho\left(\hat{\lambda}_{PLF}\right)=2\left\{\sqrt{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+3)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+3}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}}-\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+2)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}.$$

$$\hat{\lambda}_{WSELF}=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}},$$

$$\rho\left(\hat{\lambda}_{WSELF}\right)=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+2)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}-\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}}.$$

$$\hat{\lambda}_{QQLF}=\ln\left(\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{1+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}\right)^{-1},$$

$$\rho\left(\hat{\lambda}_{QQLF}\right)=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{2+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}-\left(\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{1+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}\right)^2.$$

$$\hat{\lambda}_{SLELF}=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)\exp\left(\psi(\tau+1)\right)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}},$$

$$\rho\left(\hat{\lambda}_{SLELF}\right)=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)\psi'(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}.$$

$$\hat{\lambda}_{ELF}=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}}\right\},$$

$$\rho\left(\hat{\lambda}_{ELF}\right)=\ln\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}+\psi(\tau+1)-\ln\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}\right\}.$$

The Bayes Estimators and posterior Risks under the rest of priors can be obtained in a similar manner.

4. Bayes credible interval for the doubly type ii censored data

The Bayesian credible intervals for the doubly type II censored data under informative and non-informative priors, as discussed by Saleem and Aslam (2009) are presented in the following. The credible intervals for doubly type II censored data under all priors are:

$$\frac{\chi^2_{2(\tau+1)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}<\lambda_{Uniform}<\frac{\chi^2_{2(\tau+1)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}$$

$$\frac{\chi^2_{2(\tau)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}}<\lambda_{Jeffreys}<\frac{\chi^2_{2(\tau)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}}$$

$$\frac{\chi^2_{2(\tau+1)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{m+\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{m+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}<\lambda_{Exponential}<\frac{\chi^2_{2(\tau+1)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{m+\phi_i\left(x_{(i)}\right)\right\}^{\tau+2}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{m+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}}$$

$$\frac{\chi^2_{2(\tau+a)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\phi_i\left(x_{(i)}\right)\right\}^{\tau+a+1}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\phi_i\left(x_{(i)}\right)\right\}^{\tau+a}}}<\lambda_{Gamma}<\frac{\chi^2_{2(\tau+a)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\phi_i\left(x_{(i)}\right)\right\}^{\tau+a+1}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\phi_i\left(x_{(i)}\right)\right\}^{\tau+a}}}$$

$$\frac{\chi^2_{2\left(\tau+\frac{1}{2}\right)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}^{\tau+3/2}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1/2}}}<\lambda_{In-Levy}<\frac{\chi^2_{2\left(\tau+\frac{1}{2}\right)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}^{\tau+3/2}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1/2}}}$$

5. Elicitation

Bayesian analysis elicitation of opinion is a crucial step. It helps to make it easy for us to understand what the experts believe in and what their opinions are. In statistical inference the characteristics of a certain predictive distribution proposed by an expert determine the hyperparameters of a prior distribution.

In this article, we focus on a probability elicitation method known as prior predictive elicitation. Predictive elicitation is a method for estimating hyperparameters of prior distributions by inverting corresponding prior predictive distributions. Elicitation of hyperparameter from the prior $p(\lambda)$ is conceptually difficult task because we first have to identify prior distribution and then its hyperparameters. The prior predictive distribution is used for the elicitation of the hyperparameters which is compared with the experts' judgment about this distribution and then the hyperparameters are chosen in such a way so as to make the judgment agree closely as possible with the given distribution (reader desires more detail see Grimshaw et al. (2001), Kadane (1980), O'Hagan et al. (2006), Kadane et al. (1996), Jenkinson (2005) and Leon et al. (2003)). According to Aslam (2003), the method of assessment is to compare the predictive distribution with experts' assessment about this distribution and then to choose the hyperparameters that make the assessment agree closely with the member of the family. He discusses three important methods to elicit the hyperparameters: (i) Via the Prior Predictive Probabilities (ii) Via Elicitation of the Confidence Levels (iii) Via the Predictive Mode and Confidence Level.

5.1. Prior predictive distribution

The prior predictive distribution is:

$$p(y)=\int_0^\infty p(y\mid\lambda)p(\lambda)\,d\lambda. \tag{14}$$

The predictive distribution under exponential prior is:

$$p(y)=\int_0^\infty \lambda\exp(y)\exp\left[-\lambda\left\{\exp(y)-1\right\}\right]me^{-\lambda m}\,d\lambda, \tag{15}$$

After some simplification it reduces as

$$p(y)=\frac{m\exp(y)}{\left\{m+\exp(y)-1\right\}^2},\quad y>0. \tag{16}$$

The predictive distribution under gamma prior is:

$$p(y)=\frac{ab^a\exp(y)}{\left\{b+\exp(y)-1\right\}^{a+1}},\quad y>0. \tag{17}$$

The predictive distribution under Inverse Levy prior is:

$$p(y)=\frac{\sqrt{c}\exp(y)}{2^{3/2}\left\{c/2+\exp(y)-1\right\}^{3/2}},\quad y>0. \tag{18}$$

By using the method of elicitation defined by Aslam (2003), we obtain the following hyper-parameters *m* = 0.285697, *a* = 3.49879, *b* = 0.96675 and *c* = 0.98954.

6. Predictive distribution

The predictive distribution contains the information about the independent future random observation given preceding observations. The reader desires more details can see Bolstad (2004) and Bansal (2007).

6.1. Posterior predictive distribution

The posterior predictive distribution of the future observation $y=x_{n+1}$ is

$$p(y\mid\mathbf{x})=\int_0^\infty p(\lambda\mid\mathbf{x})p(y\mid\lambda)\,d\lambda \tag{19}$$

where $p(y\mid\lambda)=\lambda\exp(x)\exp\left[-\lambda\left\{\exp(x)-1\right\}\right]$ is the future observation density and $p(\lambda\mid\mathbf{x})$ is the posterior distribution obtained by incorporating the likelihood with the respective prior distributions.

The posterior predictive distribution of the future observation $y=x_{n+1}$ under uniform prior is:

$$p(y\mid\mathbf{x})=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{(\tau+1)\exp(y)}{\left\{\phi_i\left(x_{(i)}\right)+\exp(y)-1\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}},\quad y>0.$$

The posterior predictive distribution of the future observation $y=x_{n+1}$ under Jeffreys prior is:

$$p(y\mid \mathbf{x})=\frac{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{\tau\exp(y)}{\left\{\phi_i\left(x_{(i)}\right)+\exp(y)-1\right\}^{\tau+1}}}{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{1}{\left\{\phi_i\left(x_{(i)}\right)\right\}^{\tau}}},\quad y>0.$$

The posterior predictive distribution of the future observation $y = x_{n+1}$ under exponential prior is:

$$p(y\mid \mathbf{x})=\frac{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{(\tau+1)\exp(y)}{\left\{m+\phi_i\left(x_{(i)}\right)+\exp(y)-1\right\}^{\tau+2}}}{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{1}{\left\{m+\phi_i\left(x_{(i)}\right)\right\}^{\tau+1}}},\quad y>0.$$

The posterior predictive distribution of the future observation $y = x_{n+1}$ under gamma prior is:

$$p(y\mid \mathbf{x})=\frac{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{(\tau+a)\exp(y)}{\left\{b+\phi_i\left(x_{(i)}\right)+\exp(y)-1\right\}^{\tau+a+1}}}{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{1}{\left\{b+\phi_i\left(x_{(i)}\right)\right\}^{\tau+a}}},\qquad y>0.$$

The posterior predictive distribution of the future observation $y = x_{n+1}$ under In-Levy prior is:

$$p(y\mid \mathbf{x})=\frac{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{\left(\tau+\frac{1}{2}\right)\exp(y)}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)+\exp(y)-1\right\}^{\tau+3/2}}}{\displaystyle\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}\frac{1}{\left\{\frac{c}{2}+\phi_i\left(x_{(i)}\right)\right\}^{\tau+\frac{1}{2}}}},\qquad y>0.$$

7. Simulation study

This section shows how simulation can be helpful and illuminating way to approach problems in Bayesian analysis. Bayesian problems of updating estimates can be handled easily and straight forwardly with simulation. Since we can express the distribution function of Gompertz distribution as well as its inverse in closed form, the inversion method of simulation is straightforward to implement. The study has been carried out for different values of $(n, r \text{ and } s)$ using $\lambda \in (5 \text{ and } 9)$. Censoring rate is assumed to be 20%. The estimation has been done under 10% left and 10% right censored samples. Sample size is varied to observe the effect of small and large samples on the estimators. Changes in the estimators and their risks have been determined when changing the loss function and the prior distribution of λ while keeping the sample size fixed. All these results are obtained from 5,000 Monte Carlo replications. In the Tables, the estimators for the parameter and the risk, is averaged over the total number of repetitions. Mathematica 8.0 has been used to carry out the results. All the results are summarized in the Tables 2-16.

Table 2
Bayes estimates and the posterior risks (given in parentheses) under uniform prior.

n	$\lambda=5$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	5.61695	5.71548	5.31295	4.83946	5.40453	5.21777
r=3, n-s=18	(1.76728)	(0.289531)	(0.295256)	(0.000219)	(0.060586)	(0.024384)
40	5.26997	5.34524	5.17640	4.92431	5.16326	5.18517
r=5, n-s=36	(0.77360)	(0.141628)	(0.143816)	(0.000070)	(0.030767)	(0.012176)
60	5.16861	5.25206	5.09757	4.95896	5.16041	5.061160
r=7, n-s=54	(0.495384)	(0.094223)	(0.094414)	(0.000038)	(0.020618)	(0.008113)
80	5.10433	5.18109	5.06723	4.97543	5.07710	5.05826
r=9, n-s=72	(0.360327)	(0.070083)	(0.067671)	(0.000025)	(0.015503)	(0.006063)

Table 3
Bayes estimates and the posterior risks (given in parentheses) under uniform prior.

n	$\lambda=9$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	9.96216	10.48990	9.56359	8.04300	9.54521	9.518350
r=3, n-s=18	(5.52605)	(0.53121)	(0.531474)	(6.18×10^{-6})	(0.060586)	(0.024383)
40	9.54567	9.68974	9.27813	8.48097	9.37394	9.27650
r=5, n-s=36	(2.53562)	(0.256742)	(0.257775)	(7.43×10^{-7})	(0.030767)	(0.012176)
60	9.28744	9.35120	9.17239	8.62280	9.18446	9.20005
r=7, n-s=54	(1.59601)	(0.167744)	(0.169886)	(2.47×10^{-7})	(0.020618)	(0.008113)
80	9.25833	9.26726	9.09765	8.72255	9.16633	9.11756
r=9, n-s=72	(1.19731)	(0.125579)	(0.126955)	(1.14×10^{-7})	(0.015503)	(0.006063)

Table 4
Bayes estimates and the posterior risks (given in parentheses) under jeffreys prior.

n	$\lambda=5$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	5.33537	5.45378	4.95872	4.60587	5.19675	4.95209
r=3, n-s=18	(1.07569)	(0.291008)	(0.291773)	(0.000302)	(0.064494)	(0.025599)
40	5.13786	5.21716	4.96930	4.78916	5.0666	4.95649
r=5, n-s=36	(0.75365)	(0.141998)	(0.142007)	(0.000088)	(0.030767)	(0.12469)
60	5.10566	5.19837	4.97468	4.85221	5.03535	4.95786
r=7, n-s=54	(0.49229)	(0.0950113)	(0.093877)	(0.000043)	(0.021052)	(0.008242)
80	5.06014	5.09057	4.98278	4.93334	5.02978	5.01754
r=9, n-s=72	(0.360070)	(0.069889)	(0.0640041)	(0.000027)	(0.015747)	(0.005918)

Table 5
Bayes estimates and the posterior risks (given in parentheses) under jeffreys prior.

n	$\lambda=9$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	9.57236	9.77527	8.90903	7.49634	9.26752	8.90043
r=3, n-s=18	(5.40728)	(0.549732)	(0.528919)	(0.000012)	(0.064494)	(0.025599)
40	9.46732	9.41713	8.92703	8.17871	9.12322	8.98946

r=5, n-s=36	(2.40182)	(0.258308)	(0.255110)	(1.13×10^{-6})	(0.030767)	(0.012469)
60	9.19865	9.26592	8.95093	8.45766	9.09228	9.01250
r=7, n-s=54	(1.59832)	(0.169270)	(0.168913)	(3.46×10^{-7})	(0.021052)	(0.008242)
80	9.06331	9.17781	8.99599	8.57278	9.03260	8.99562
r=9, n-s=72	(1.14973)	(0.139274)	(0.117009)	(1.52×10^{-7})	(0.015747)	(0.005918)

Table 6
Bayes estimates and the posterior risks (given in parentheses) under exponential prior.

n	$\lambda=5$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	5.14592	5.19746	4.88107	4.46527	4.93563	4.83442
r=3, n-s=18	(1.46631)	(0.263280)	(0.271242)	(0.000313)	(0.060586)	(0.024381)
40	5.11438	5.11754	4.94468	4.67774	4.97820	4.93158
r=5, n-s=36	(0.727349)	(0.135950)	(0.137376)	(0.000093)	(0.030767)	(0.012176)
60	5.04997	5.10130	4.97061	4.81099	4.99201	4.95522
r=7, n-s=54	(0.471899)	(0.091520)	(0.092063)	(0.000045)	(0.020618)	(0.008113)
80	5.02497	5.07341	4.98150	4.88216	4.99588	4.97235
r=9, n-s=72	(0.353406)	(0.068756)	(0.063702)	(0.000028)	(0.015503)	(0.006099)

Table 7
Bayes estimates and the posterior risks (given in parentheses) under exponential prior.

n	$\lambda=9$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	8.68498	9.09142	8.31611	7.10907	8.440990	8.31475
r=3, n-s=18	(4.16003)	(0.451528)	(0.462112)	(0.000013)	(0.060586)	(0.024381)
40	8.84439	9.05936	8.53135	7.90988	8.72109	8.62070
r=5, n-s=36	(2.16995)	(0.239504)	(0.237022)	(1.33×10^{-6})	(0.030767)	(0.012176)
60	8.91833	9.04953	8.70212	8.20969	8.77526	8.75455
r=7, n-s=54	(1.47164)	(0.162349)	(0.161173)	(4.05×10^{-7})	(0.020618)	(0.008113)
80	8.94251	9.04509	8.80830	8.49391	8.86013	8.83115
r=9, n-s=72	(0.957981)	(0.120453)	(0.112143)	(1.76×10^{-7})	(0.015503)	(0.006099)

Table 8
Bayes estimates and the posterior risks (given in parentheses) under gamma prior.

n	$\lambda=5$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	4.86038	4.92933	4.67084	4.36813	4.69282	4.62331
r=3, n-s=18	(1.14488)	(0.221630)	(0.227908)	(0.000267)	(0.052623)	(0.021779)
40	4.92062	4.95329	4.79951	4.66174	4.84772	4.80936
r=5, n-s=36	(0.627488)	(0.123091)	(0.124686)	(0.000084)	(0.028571)	(0.011496)
60	4.95922	4.96693	4.87245	4.77996	4.87694	4.90818
r=7, n-s=54	(0.434842)	(0.085284)	(0.086254)	(0.000043)	(0.019608)	(0.007807)
80	4.96765	5.00488	4.90104	4.82579	4.94684	4.92166
r=9, n-s=72	(0.329846)	(0.065699)	(0.062742)	(0.000028)	(0.014927)	(0.005925)

Table 9
Bayes estimates and the posterior risks (given in parentheses) under gamma prior.

n	$\lambda=9$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	7.43222	7.59738	7.14111	6.37941	7.26513	7.11246
r=3, n-s=18	(2.63516)	(0.34157)	(0.34842)	(0.000019)	(0.052623)	(0.021776)

40	8.12568	8.20407	7.93131	7.35221	7.97027	7.86598
r=5, n-s=36	(1.70386)	(0.203869)	(0.206041)	(2.07×10^{-6})	(0.028571)	(0.011496)
60	8.34277	8.47785	8.23122	7.7659	8.31594	8.20961
r=7, n-s=54	(1.22850)	(0.145563)	(0.145706)	(6.18×10^{-7})	(0.019608)	(0.007807)
80	8.51335	8.51927	8.51960	8.40357	8.54807	8.49527
r=9, n-s=72	(0.967644)	(0.114574)	(0.112076)	(2.52×10^{-7})	(0.014927)	(0.005925)

Table 10
Bayes estimates and the posterior risks (given in parentheses) under in-levy prior.

n	$\lambda=5$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	4.67626	4.8 0701	4.42247	4.20880	4.56377	4.44211
r=3, n-s=18	(1.23768)	(0.249815)	(0.252769)	(0.000414)	(0.062480)	(0.024973)
40	4.85201	4.92341	4.68724	4.51897	4.84027	4.71898
r=5, n-s=36	(0.66105)	(0.132201)	(0.132057)	(0.000117)	(0.031248)	(0.012321)
60	4.87461	4.96465	4.80403	4.67936	4.87159	4.82525
r=7, n-s=54	(0.443122)	(0.089873)	(0.089808)	(0.000054)	(0.020833)	(0.008177)
80	4.91181	4.97676	4.87052	4.73760	4.87924	4.87725
r=9, n-s=72	(0.337039)	(0.068185)	(0.069719)	(0.000033)	(0.015625)	(0.006118)

Table 11
Bayes estimates and the posterior risks (given in parentheses) under in-levy prior.

n	$\lambda=9$					
	SELF	**PLF**	**WSELF**	**QQLF**	**SLELF**	**ELF**
20	7.64419	7.84752	7.33163	6.38845	7.44037	7.24521
r=3, n-s=18	(3.27333)	(0.407808)	(0.419023)	(0.000028)	(0.062480)	(0.024973)
40	8.26884	8.39473	8.10603	7.41991	8.22511	8.08924
r=5, n-s=36	(1.91314)	(0.225406)	(0.228374)	(2.45×10^{-6})	(0.031248)	(0.012321)
60	8.54645	8.56546	8.41949	7.94316	8.46694	8.35261
r=7, n-s=54	(1.36176)	(0.155057)	(0.157394)	(5.66×10^{-7})	(0.020833)	(0.008177)
80	8.65756	8.74001	8.53004	8.35445	8.60776	8.57610
r=9, n-s=72	(1.044985)	(0.121042)	(0.120318)	(2.35×10^{-7})	(0.015625)	(0.006118)

Table 12
The lower (LL), the upper (UL) and the width of the 95% CI under uniform prior.

r ,n, n-s	$\lambda=5$		**Width**	$\lambda=9$		**Width**
	LL	**UL**		**LL**	**UL**	
3, 20, 18	3.15183	8.26949	5.11766	5.63399	14.78197	9.14798
5,40, 36	3.53151	7.02306	3.49155	6.38894	12.70562	6.31668
7, 60, 54	3.76513	6.61003	2.84490	7.11693	12.49443	5.37750
9, 80, 72	3.91623	6.38007	2.46384	7.00353	11.40977	4.40620

Table 13
The lower (LL), the upper (UL) and the width of the 95% CI under jeffreys prior.

r ,n, n-s	$\lambda=5$		**Width**	$\lambda=9$		**Width**
	LL	**UL**		**LL**	**UL**	
3, 20, 18	2.92981	7.92574	4.99593	5.23712	14.16749	8.93037
5,40, 36	3.41432	6.86390	3.44958	6.17694	12.41768	6.24074
7, 60, 54	3.68424	6.50625	2.82201	6.96407	12.29831	5.33424
9, 80, 72	3.85449	6.30358	2.44909	6.89277	11.27233	4.37956

Table 14
The lower (LL), the upper (UL) and the width of the 95% CI under exponential prior.

r ,n, n-s	$\lambda = 5$		Width	$\lambda = 9$		Width
	LL	**UL**		**LL**	**UL**	
3, 20, 18	2.91469	7.64731	4.73262	4.91867	12.90516	7.98649
5,40, 36	3.39692	6.75541	3.35849	5.96191	11.85638	5.89447
7, 60, 54	3.66813	6.43973	2.77160	6.77819	11.89974	5.12155
9, 80, 72	3.83995	6.25581	2.41586	6.76340	11.01851	4.25511

Table 15
The lower (LL), the upper (UL) and the width of the 95% CI under gamma prior.

r ,n, n-s	$\lambda = 3.5$		Width	$\lambda = 7$		Width
	LL	**UL**		**LL**	**UL**	
3, 20, 18	2.91155	7.15406	4.24251	4.44826	10.92998	6.48172
5,40, 36	3.37322	6.54277	3.16955	5.56992	10.80353	5.23361
7, 60, 54	3.64179	6.30489	2.66310	6.41487	11.10580	4.69093
9, 80, 72	3.81482	6.15804	2.34322	6.49959	10.49191	3.99232

Table 16
The lower (LL), the upper (UL) and the width of the 95% CI under Inverse Levy Prior.

r ,n, n-s	$\lambda = 3.5$		Width	$\lambda = 7$		Width
	LL	**UL**		**LL**	**UL**	
3, 20, 18	2.66511	7.09772	4.43261	4.34171	11.56283	7.22112
5,40, 36	3.24988	6.49766	3.24778	5.58917	11.17473	5.58556
7, 60, 54	3.56158	6.27096	2.70938	6.47964	11.40887	4.92923
9, 80, 72	3.75949	6.13637	2.37688	6.55129	10.69325	4.14196

8. Conclusion

The simulation study has displayed some interesting properties of the Bayes estimates. After an extensive study of results, conclusions are drawn regarding the behavior of the estimators. The risks of the estimates seem to be large in case when the value of the parameter is large and small for relative smaller value of the parameter except under quasi-quadratic loss function. However, the risks under said loss functions are reduced as the sample size increases. Another interesting remark concerning the risks of the estimates is that increasing (decreasing) the value of the parameter reduces (increases) the risks of the estimates under quasi-quadratic loss function. The performance of squared-log error loss function and entropy loss function is independent of choice of parametric value. The above study depicts that the estimated value of the parameter converges to the true value of the parameter by increasing the sample size. The greater values of the parameter impose a negative impact on convergence and performance of the estimates. The effect of the increasing values of the parameter is in the form of underestimation assuming each informative prior. The patterns of the estimates discussed above, are almost similar under uniform and Jeffreys priors. However, the performance of the uniform prior is better for estimates under SLELF, ELF, PLF and QQLF. While for estimates, under SELF and WSELF, the performance of the Jeffreys prior is better than uniform prior. In comparison of informative priors, the gamma prior provides the better estimates as the corresponding risks are least under said loss functions with few exceptions. While the exponential prior turns out to perform better under QQLF for larger values of the parameter, therefore it produces more efficient estimates as compared to other informative priors.

After an extensive study of the results, thus obtained, we observed that the risks of the estimators under doubly type II censored data assuming uniform prior behave similarly to the risks of the estimators under exponential prior under SLELF and ELF. In addition, estimates under quasi-quadratic loss function give the minimum risks among all loss functions for each prior. The Credible interval are in accordance with the point estimates, that is, the width of credible interval is inversely proportional to sample size while, it is directly proportional to the parametric value. From the Table 12-16, appended above, reveal that the effect of the parametric values in the form of larger width of interval. The Credible interval assuming gamma prior is much narrower than the credible intervals assuming informative and non-informative priors. It is the use of prior information that makes a difference in terms of gain in precision. The study can further be extended by considering generalized versions of the distribution under variety of circumstances.

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