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Investigation of 3-D friction coefficient in pinned composite plates

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ABSTRACT

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Connecting composite materials to each other is of great importance. Mechanical joints are widely applied in industries; therefore they are of more attention in designers view. In this kind of joint, determinations of ultimate strength, type of rupture and stress analysis at pin location are vital. The aim of this paper is to study interlaminar stress in mechanical joints and the effect of different stacking sequences on generated stresses between layers and their stress distribution. The effect of friction on stresses between layers in composite plates is also investigated. The problem was modeled three dimensionally in ABAQUS software, considering interlaminar stresses, friction and different stacking sequence. As a result, some remarks are presented; for example: When sequence is symmetry, stress field is symmetry toward middle plate, too. In cross-ply layup, stress fields are symmetry toward bearing and mid-plane. Applying friction in different directions, it was seen that friction coefficient in θ direction was higher and more important than in other directions. As a result, radial and tangential stress diagrams for two condition of friction in z direction and without friction were so close and also stress diagrams for two condition of friction μ_{θ} and $\mu_{\theta,z}$ have more similarity.

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1. Introduction

Most of machines are composed of members which are connected by mechanical joints. The most important duty of joints is transferring load between different parts. These joints could be temporary or permanent. Temporary joints are used only once; although they are cheaper, more resistant and tolerate more dynamic loads. Conversely, permanent joints can be used for long times; though they are not a good choice for transferring loads.

Connecting composite materials to each other is of great importance. In general, there are three major categories in composite plate joints: adhesive joints, mechanical joints and a mixture of adhesive and mechanical joints. Mechanical joints are widely applied in industries; therefore they are of more attention in designers view. In this kind of joint, determinations of ultimate strength, type of rupture and stress analysis at pin location are vital.

In recent years with the development in computers hardware and software, the use of numerical methods especially Finite Element Method (FEM) and FEM-based softwares are applied more and more. By using FEM, a designer can model and analyze complex geometries, different loadings and Boundary Conditions (BCs) in these kinds of joints; while this is almost unreachable by analytic methods.

In the past three decades, numerous papers have been revealed in mechanical joint fields. Most of these papers were composed of experimental results which investigate the effect of temperature, moisture, stacking sequence, geometry, tolerance and wobble between pin and hole [Hyer and Liu, 1984, Kelly and Hallström, 2004]. Only a small part of research in the area of mechanical joints dedicated to analytic methods. In these works, 2-D models with simplifications were used and stress field were determined around hole [McCarthy, 2005a, McCarthy et al., 2005b]. In other papers, stress field in mechanical joints were anticipated using numerical methods [Aktas and Dirikolu, 2004, Iyer, 2001].

Mechanical behavior of joined composite plates compared to other plates is much complicated. "Camanho and Matio" (1997) had done comprehensive study on the failure of composite plates around the hole. Besides, among this study, numerous simple models offered for numerical analysis of stress zone and failure in mechanical joints of composite plates. For example Dano et al. (2000) investigated 2D model to predict behavior of rigid joints between loaded composite plates. In this regard, some 3D analysis was done by Chen et al. (1995) and Ireman (1998). Ireman (1998) assumed joints was elastic and ignored effect of friction between composite plates.

Yang et al. (2003) defined exact numerical method for mechanical behavior of composite plates with elastic joints. They assumed isotropic joints and also considered friction in contact surface between joints and plates. In their analysis, plates were assumed symmetric. Hyer and Liu (1984) investigated effect of pin elasticity, clearance and friction on radial and tangential stress distribution around hole in mechanical joints of orthotropic plates by numerical method. Their model was 2D and constituted of two symmetric layers to investigate aforementioned parameters.

Chen et al. (1995) analyzed 3D contact stress in the mechanical joints of symmetric composite material layers by numerical method. Their results were accomplished using local contact between pin and hole, considering 3D friction and clearance.

Xiao et al. (2000) studied effective friction coefficient for the layered composite materials in their analysis of plates with pin joints. They illustrated that in different directions, friction coefficient had different values in one layer. Thus they used non-uniform distribution of friction coefficient as a function of angle, for their modeling. Using finite element model, Iyer (2001) obtained distribution of radial and tangential stress in pin joints and checked friction, geometry and two axial loading effects. His research was limited to isotropic plates and he investigated the effect of pin materials on radial and tangential stress distribution.

Kadivar and Shahi (2002) calculated contact surfaces between pin and hole along thickness of multiple layers by 3D finite element model with ANSYS software. They assumed solid pin. Considering appropriate boundary conditions, they also modeled pin-hole effect and checked geometry effect on contact region and stress distribution.

McCarthy et al. (2006) investigated clearance effects on stress distribution for multiple pin joint, by use of FEM and MARC software. Yavari and Kadivar (2007) studied the effects of contact surface between pin and shell in mechanical single edge "composite" materials by use of ABAQUS software. In this research, they considered 3D joint behavior and also friction effect. Wimmer and Pettermann (2008) numerically simulated separation in composite plates. In his study, he used combination of critical and failure loadings. Alenfaie (2009) modeled composite plates with internal separation plates by use of finite element method. He used a 3D model and calculated natural frequency and displacement in various cases.

The aim of this paper is to study interlaminar stress in mechanical joints and the effect of different stacking sequences on generated stresses between layers and their stress distribution. The effect of friction on stresses between layers in composite plates is also investigated.

In order to solve the problem, composite joints are modeled three dimensionally in ABAQUS 6.9.1 software, considering interlaminar stresses and friction between layers. Solid elements were used for composite plates. After verifying of the software results, effect of several parameters on interlaminar stresses and stress distribution are checked. This procedure is applied to different cases of symmetric to symmetric, asymmetric to symmetric and asymmetric to asymmetric. In each case, the effect of quasi-isotropic, cross-ply and angle-ply lay-ups on stress between plates, stress distribution, maximum stress and its location are investigated.

2. Problem description

The base model used in this study is a rectangular plate of 155 mm length, 48 mm width and 52 mm thickness. An 8 mm diameter hole was embedded by 24mm distance from edge (Fig1). Contact elements were defined between pin and composite plate. A distributed load of 20 MPa was imposed on the edge of composite plate, in the X-direction. As it was mentioned, pin was considered elastic which fitted in the plate hole. Pin specification is illustrated in Table 1. The composite plate has 8 layers and is totally 5.2 millimeters thick.

Table1
Plate and joint dimensions.

L(mm)	W(mm)	t(mm)	D(mm)	e(mm)
155	48	5.2	8	24

Boundary conditions were applied on pin.

Joint geometry was defined by ASTM standard (D5961 M-96). Pin geometry ratio is $w/d = 6$, $e/d = 3$, $d/t = 1.6$, as a result it has high strength. Geometry and elastic properties of pin which is made of titanium alloys is illustrated in Table 2.

Table 2
Pin geometry and elastic properties.

w/d	e/d	d/t	E_b (GPa)	ν_b
6	3	1.6	110	0.29

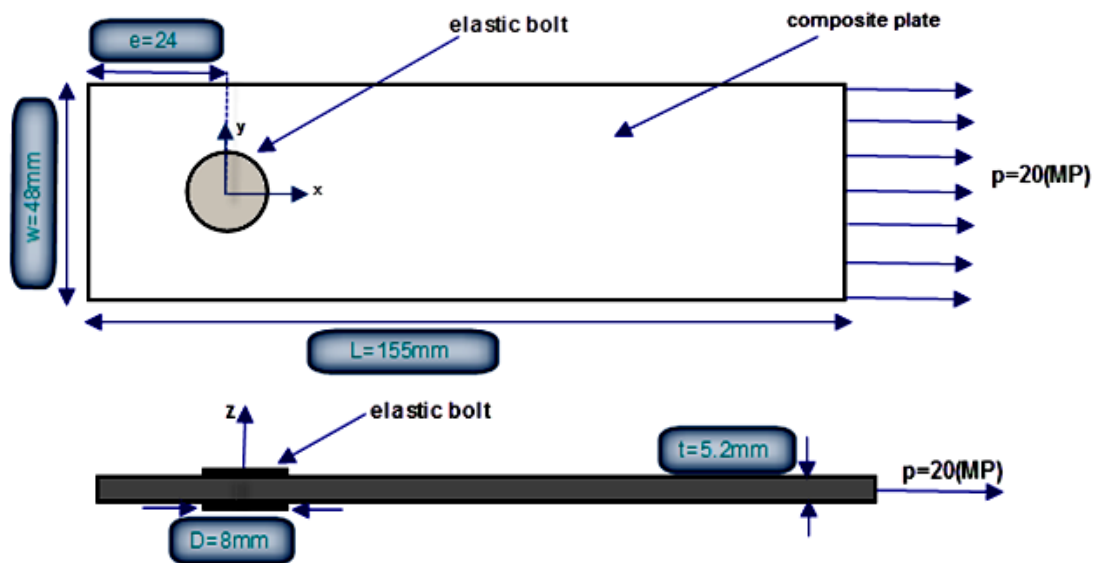


Fig. 1. Composite plate geometry and joints.

Material of each composite plate layer is Carbon-Epoxy that is recently used in space structures such as HTA/6376 Carbon-Epoxy, because of their very high strength. Elastic properties of each composite plate layer are illustrated in Table 3.

Table 3
Elastic properties of each composite plate layer

E_{11} (GPa)	E_{22} (GPa)	E_{33} (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
140	10	10	5.2	5.2	3.9	0.3	0.3	0.3

3. Finite element mesh

C3D8 element, which is a linear 3D solid element, was used in modeling both plate and pin. Properties of each portion were assigned according to tables 1, 2 and 3. As mentioned before, contact element was considered between plate and pin. Since there was localized stress field around the hole and accuracy was needed, fine mesh were used in this zone. But in order to decrease computation time and processing cost, a larger mesh was used far from hole.

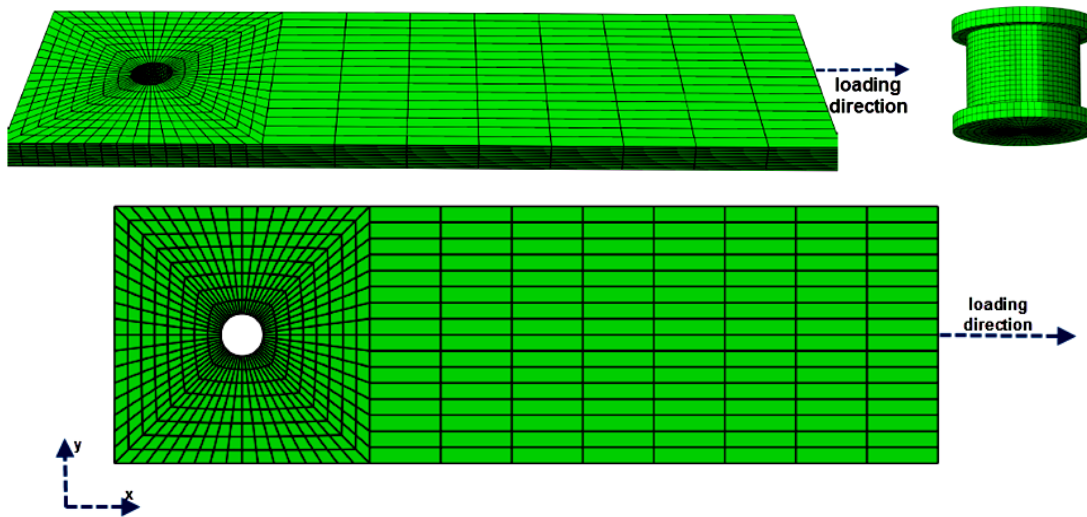


Fig. 2. Finite element model.

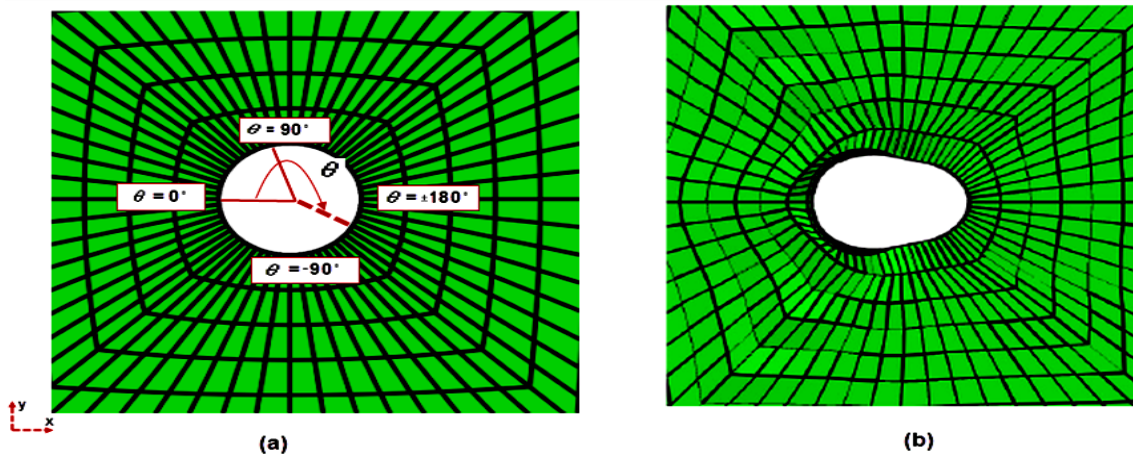


Fig. 3. Plate deformation and the coordinate used; a) Before loading b) After loading.

A linear analysis was adopted for solving the problem, because the applied load amount caused very small displacement. Figure 2 shows how plate and pin were meshed.

Plate was loaded in x direction and two ends of pin were fixed. As a result of this loading, hole was deformed (Fig. 2). According to this, the specified coordinate (illustrated in Fig. 3) was used to present results.

3. Results and discussion

Because of differing stacking sequences in composite plate, the direction of fibers were not the same as the direction of loading; thus in each layer, the stress between layers was different. In each layer, fibers angle was considered by positive direction of X-coordinate and Z-coordinate was assumed along thickness.

3.1. Stress field in quasi-isotropic layup $[0/45/90/-45]$

The first point in this stacking sequence is asymmetric stress field according to bearing plane, because of 45° and -45° layers. Instead, because of symmetric stacking sequence, stress field is symmetric according to middle plane.

Figure 4 shows normalized radial stress vs. the specified coordinate θ . In each layer, the maximum radial stress appears in a region that fibers are perpendicular to contact surface. The reason is that, stiffness of these regions is higher, as expected. The other important result is that the maximum radial stress occurs in 0° layers because the fibers is in the direction of applied load, and has much higher stiffness than the other layers; therefore the radial stresses in 90° layers is the lowest.

Investigating the effect of adjacent layers stress field on each layer, it was expected that in 0° and 90° layers that stress fields are symmetric (in 0° and 90° layers, the direction of load and layer fibers are parallel and perpendicular, respectively), but it was found that they are not. This was because of existence of 45° and -45° layers with asymmetric stress fields, which influenced on adjacent layers and caused asymmetric field in 0° and 90° layers. This showed that symmetric modeling of composite plates about bearing plane was not correct and was erroneous; thus complete 3D modeling was considered in all lay-ups. Figure 4 illustrates radial stress curves between layers.

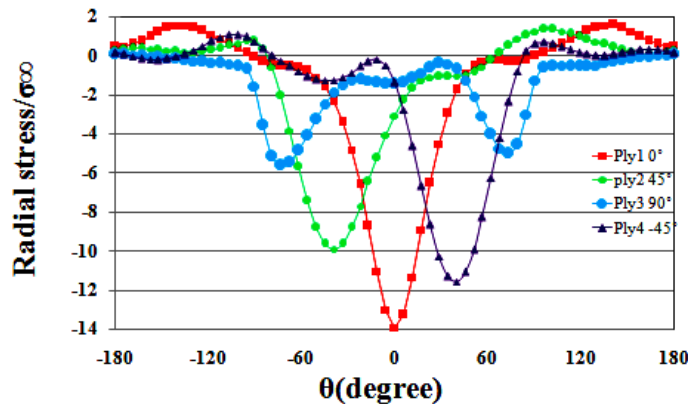


Fig. 4. Radial stress comparison between layers in Quasi-isotropic lay-up $[0/45/90/-45]$.

Tangential stresses, however, have their maximum values in the contact surface region. Specially the maximum stress values appears in 0° layers, because of high stiffness of this region, and the minimum in 90° layers, because of the direction of fibers and applied load that cause low stiffness in these layers (Fig 5).

In order to non-dimension the stresses; they were divided by an average value of bearing stress, which was obtained by following equation:

$$\sigma_\infty = \frac{P}{Dt}$$

In this work, its value is equal to 20 MPa.

Maximum stresses, after 0° layers, occurred in 45° and -45° layers. Despite their similar fiber direction relative to the direction of the applied force, stresses in both layers were not equal. The reason is that their positions in thickness direction are different and also the presence of adjacent layers caused this asymmetry. An interesting point was maximum stress in 45° layers which was exactly located in the angle of 40° and in -45° layers located in the angle of -40° which had 5° rotation compared to the fiber direction. This shows the effect of adjacent layers too. Another important point is that the closer the angles of adjacent layers, the less difference between stress values.

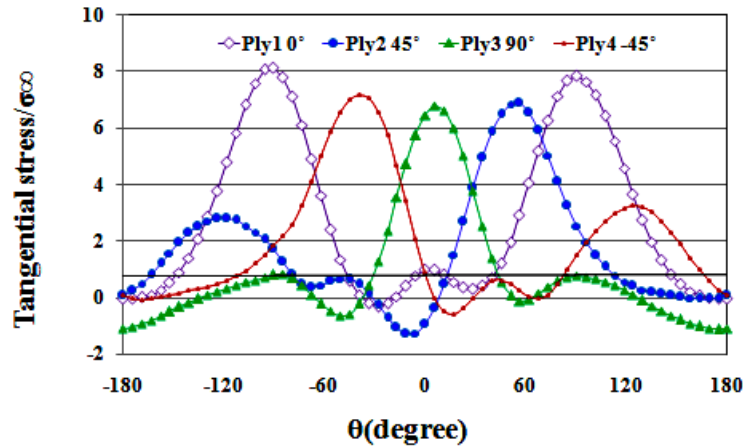


Fig. 5. Tangential stress comparison between layers in Quasi-isotropic layup $[0/45/90/-45]_s$.

Stress field in cross-ply layup $[0/90/0/90]_s$,

In this layup the first point of interest is symmetry in stacking sequence, which causes symmetric stress field. The important point about this layup is that fibers and load directions are symmetric; however this is not true for quasi-isotropic layup as mentioned in the previous section.

The maximum value of stress appears in 0° layers; because the directions of fibers and applied load are the same. Similarly in 90° layers the minimum value of stress exists because fibers direction is perpendicular to applied load direction. This is well demonstrated in Fig 6.

Tangential stresses in each layer are maximum in the region that fibers are tangent to contact surface. Because of more stiffness of 0° layer, stress in this layers are more and similarly 90° layers have least stresses because of less stiffness in these layers.

Comparison between tangential stresses in Fig 7 shows that in first layer (0°) maximum tangential stresses (MTSs) occur in angle of 90° and -90°, in second layer (90°) maximum stress occurs in angle of 0°. These diagrams confirm the investigated results. As expected, maximum stresses occur in the region that fiber directions are tangent to contact surface. Meanwhile tangential stresses in 0° layers are greater because of high stiffness of these layers.

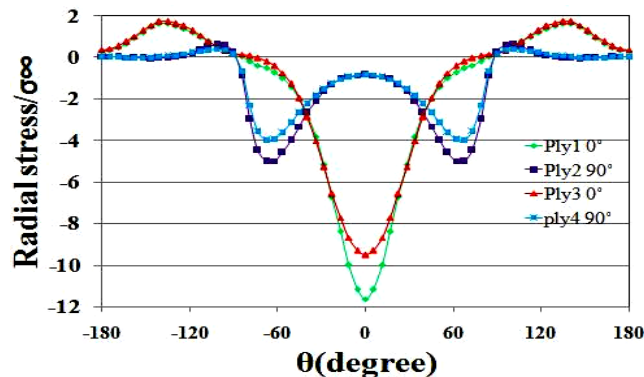


Fig. 6. Radial stresses comparison between layers in cross-ply layup $[0/90/0/90]_s$.

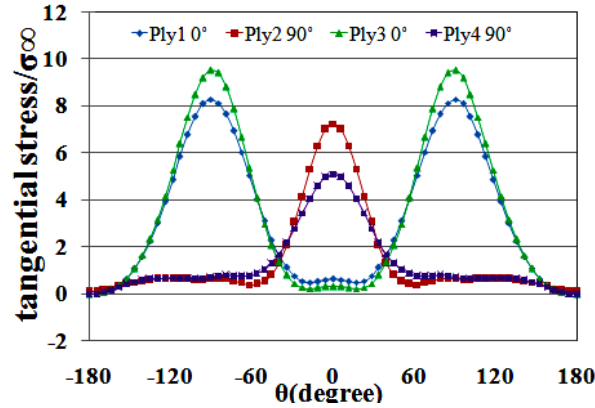


Fig. 7. Tangential stress comparison between layers in cross-ply layup $[0/90/0/90]$.

3.2. Stress field in angle-ply layup $[45/-45/45/-45]$,

This stacking sequence is also symmetric, which causes symmetric stress field according to mid-plane. Since fibers direction does not coincide with load direction, stress field is not symmetric according bearing plane.

As expected, the stiffness in first and third layers is greatest, because fibers direction is perpendicular to contact surface and as a result, maximum radial stress occurs there. Though, as mentioned before, because of adjacent layers, maximum stress does not occur in 45° . Similarly in the second and fourth layers, maximum value of radial stress occurs near angle of 45° . In order to see the radial stresses in layers accurately, radial stress diagram in the first four layers are studied and illustrated in Fig 8. According to radial stress diagrams in first and third layers, maximum stresses occur in $\pm 39^\circ$. Location of maximum stresses is about 6° different with $\pm 45^\circ$ which shows the effect of adjacent layers. Similarly in second and fourth layers, maximum radial stresses (MRS) appear at 39° .

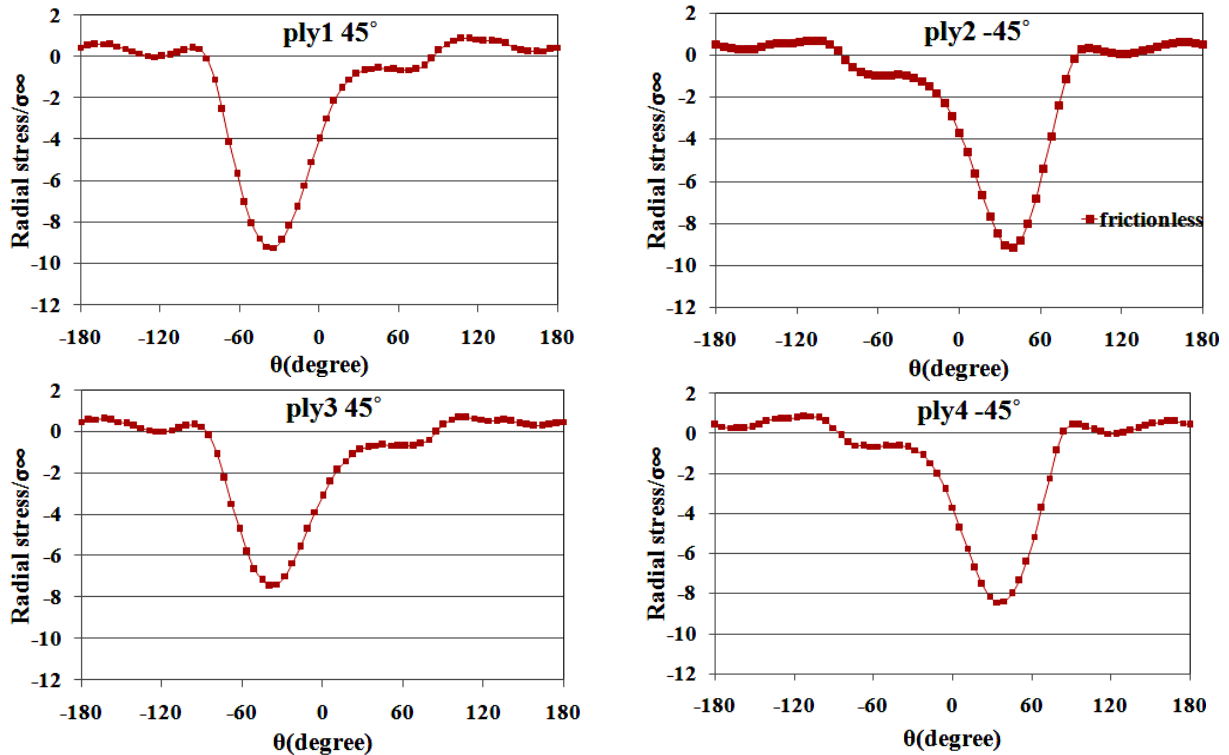


Fig. 8. Tangential stresses comparison between layers in angle-ply layup $[45/-45/45/-45]$.

Tangential stresses in region that fibers are tangent to surface contact have the highest values. In 45° layer at angle of 45°, fibers are tangent to contact surface and cause MTSs in this region. Athwart our expectation, the location of maximum stress does not coincide exactly with the angle of each layer in -45°, because of adjacent layer effect. The exact location of the maximum tangential stress and its values is shown in Fig 9.

Maximum tangential stress in first and third layers occurs in angle of 50°. Similarly in second and fourth layers, MTSs occur in -50°.

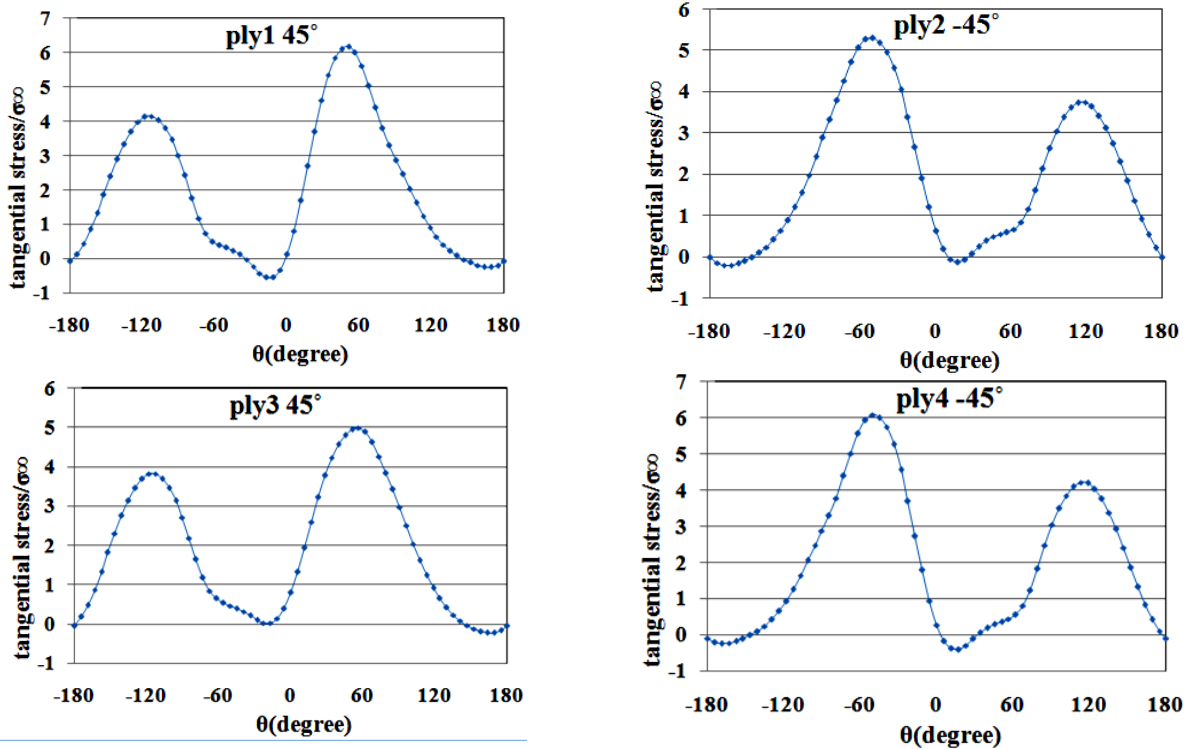


Fig. 9. Tangential stresses comparison between layers in angle-ply layup $[45/-45/45/-45]_s$.

3.3. Effect of friction between pin and plate

In this section, stress field is investigated considering friction between contact surfaces. Since the goal of this study is 3-D analysis, the problem was modeled three dimensional in previous sections (without coefficient of friction). A very important point in the present study, which will be discussed in two sections, is that the considered friction coefficient is three dimensional and varies based on fibers direction in each layer, number of layers and their thicknesses. As will be shown, friction coefficient is an effective coefficient.

3-D friction coefficient was derived according to the work done by Xiao et al. (2000). They obtained 2-D effective friction coefficient of a laminate pin-loaded composite plate. Following their study, the coefficient for 3-D friction was derived and the value of friction coefficient in desired directions was determined.

3.4. Stress field in cross-ply layup $[0/90/0/90]_s$

Friction factor between pin and plate in cross-ply layup was calculated. It was equal to 0.2179 and 0.2 in tangential and thickness direction, respectively. For further evaluation of friction effect, we applied a force to the plate in three conditions and compare them. In first condition, friction was considered only in z direction, in second condition in θ direction and finally in both z and θ direction. What is investigated at first is radial stress (Fig 10).

In cross-ply layup with friction, stress field is symmetric too. This is because of symmetry between fibers and load direction. In all conditions maximum amount of stress in first and third layers occur in angle of 0°; because at this angle, fibers are perpendicular to surface of hole and bolt and stiffness of plate is maximum. According to the abovementioned reason, in second and fourth layers, maximum stresses are seen in angle $\pm 67.7^\circ$. As expected, the

value of radial stresses in 0° layers is more than in 90° layers. The main reason is that the direction of fibers and loads are parallel.

A very interesting point in the above and other diagrams is that the friction coefficient in θ direction is more important than in z direction. Because of this reason, radial stress diagrams with and without considering μ_z are so close in each layer and stress diagram for μ_θ and $\mu_{\theta,z}$ are similar too.

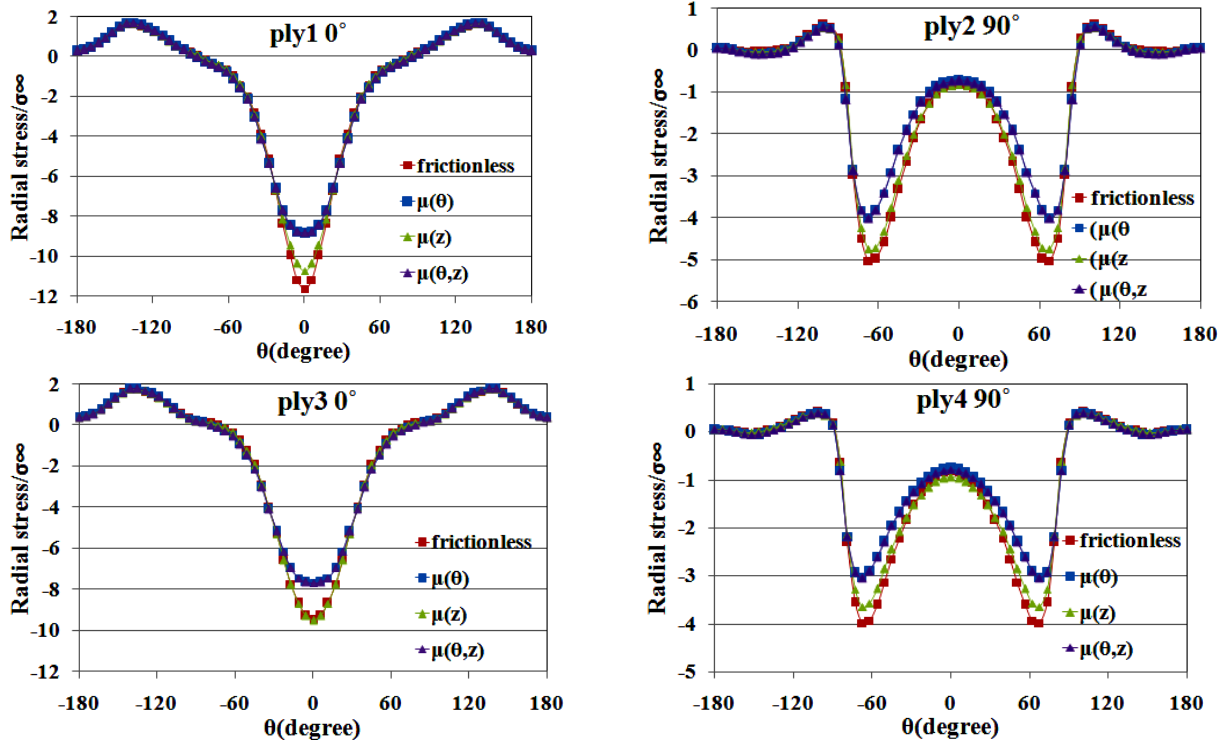


Fig. 10. Radial stress comparison between layers in cross-ply layout $[0/90/0/90]_s$.

According to Fig 10, it is seen that in first layer (0°) maximum difference in MRSs for $\mu = 0$ and μ_z is 8.8% and for μ_θ and $\mu_{\theta,z}$ is 0.5%. These values show dominant friction effect in θ direction. Also in this layer, the difference between maximum and minimum of MRSs, for $\mu = 0$ and $\mu_{\theta,z}$ is 32%. This difference in third layer (0°) is less. Under two situations $\mu = 0$ and μ_z , MRSs are equal and no difference is observed; this matter is also true for μ_θ and $\mu_{\theta,z}$. In third layer, the difference between maximum and minimum of MRSs, for $\mu = 0$ and $\mu_{\theta,z}$ is 24%. In second and fourth layers, since fiber and load directions are perpendicular, the magnitude of radial stress is minimum. In second layer (90°) difference between MRSs for $\mu = 0$ and μ_z is equal to 5%; while for μ_θ and $\mu_{\theta,z}$ it is equal to 0.12%. In this layer the difference between maximum and minimum of MRSs, for $\mu = 0$ and $\mu_{\theta,z}$ is 25%. Similarly in fourth layer (90°) maximum radial stress for $\mu = 0$ and μ_z and also for μ_θ and $\mu_{\theta,z}$ are same. Meanwhile the difference between maximum and minimum of MRSs, for $\mu = 0$ and $\mu_{\theta,z}$ is 30%.

As expected, because of symmetric layup and symmetry in fiber and load direction in all layers, radial stresses diagrams are quite symmetric in each layer. By the way, considering friction in this layup does not affect on maximum stress location.

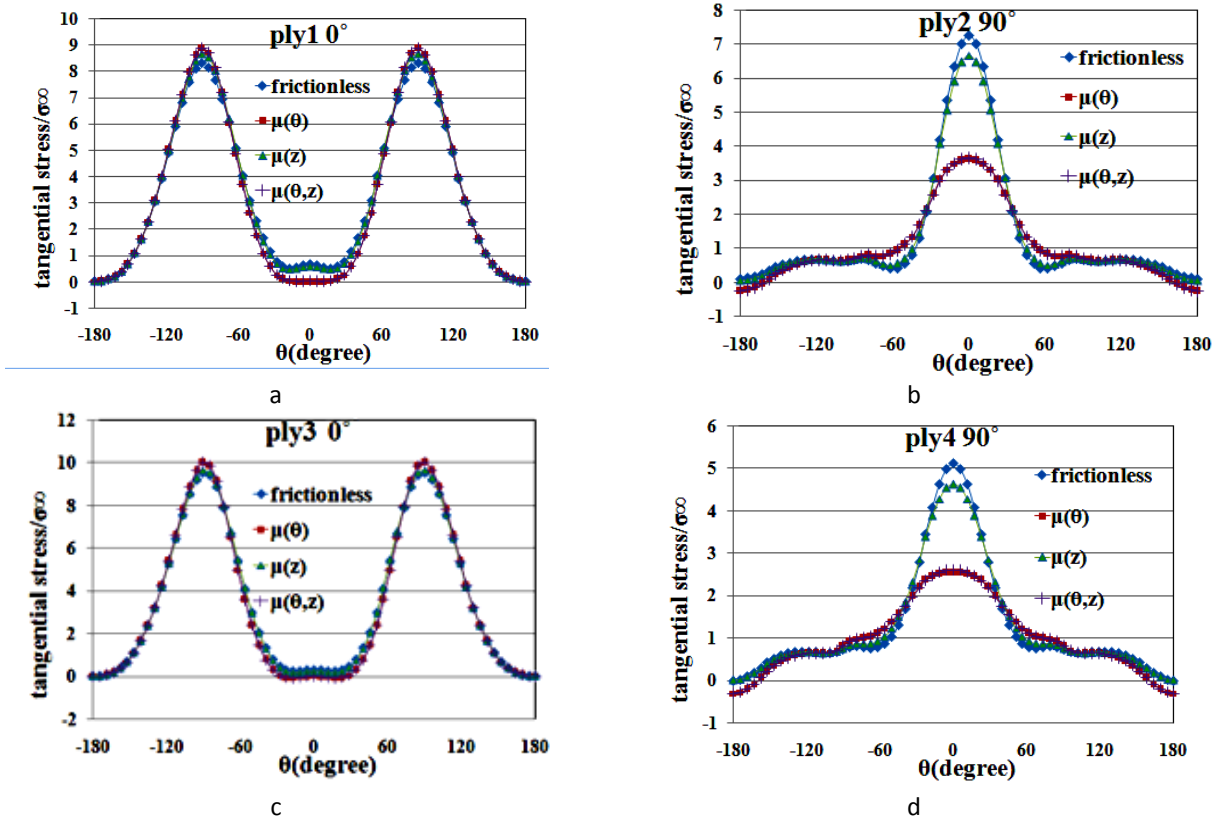


Fig. 11. Tangential stresses comparison between layers in cross-ply layup $[0/90/0/90]$.

a) 0° layer b) 90° layer c) 0° layer d) 90° layer

In Fig 11, tangential stresses are illustrated. According to this diagrams it can be seen that in first layer (0°) maximum difference of MTSs for $\mu = 0$ and μ_z is 4% but for μ_θ and $\mu_{\theta,z}$ there are no difference between MTSs. It is also observed that the difference of MTSs between μ_θ and $\mu = 0$ is about 6%. For third layer (0°), however, these values equal 1% for $\mu = 0$ and μ_z , 0% for μ_θ and $\mu_{\theta,z}$ and 2% for μ_θ and $\mu = 0$. These values for second and forth layer (90°) are 9% and 7% for $\mu = 0$ and μ_z , 1% and 2.5% for μ_θ and $\mu_{\theta,z}$ and 50%, respectively.

Similar to radial stresses, the location of MTSs in all layers and for all conditions is symmetric. In first and third layer (0°), fibers are tangent to contact surface at angle of 90°, thus MTS occurs in this angle. Similarly in second and fourth layers (90°), MTS locates at angle of 0°.

Radial displacement is demonstrated in Fig 13. Taking a glance on this diagram, it is shown that at contact surface, friction has no significant effect on radial displacement

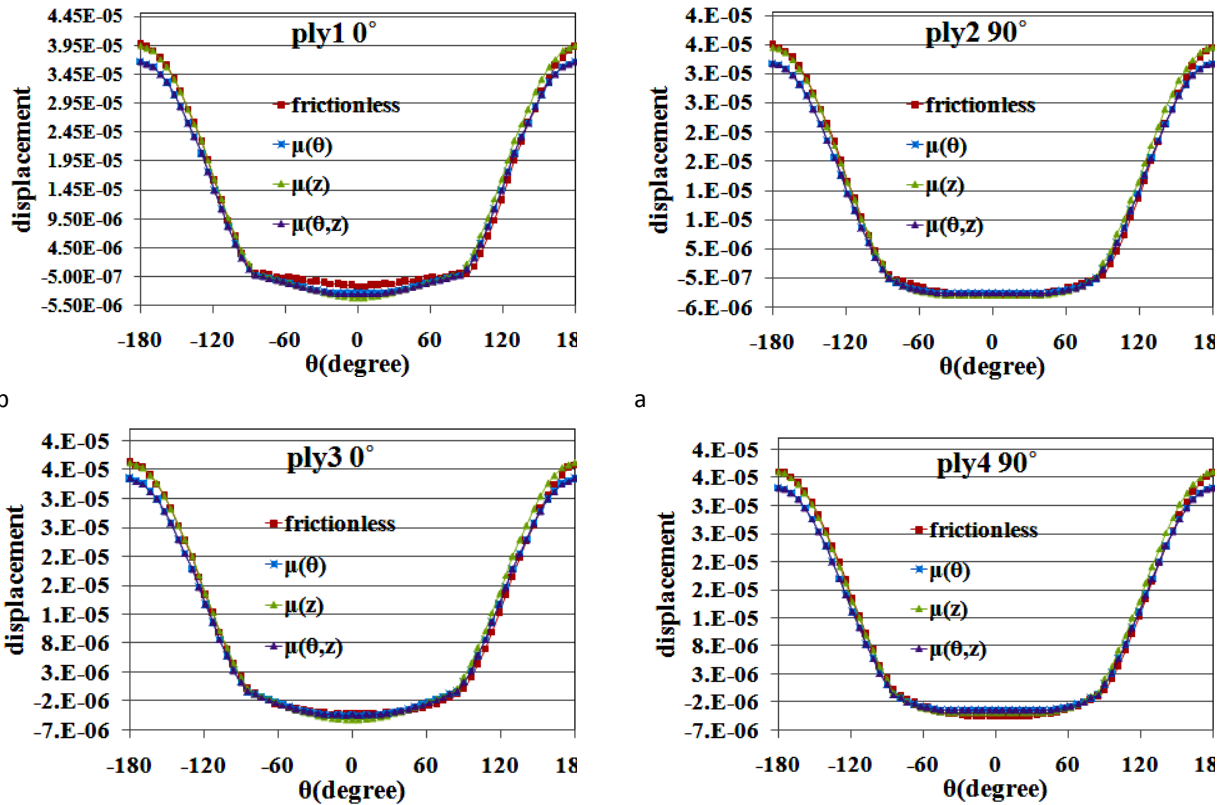


Fig. 13. Radial displacement comparison between layers in cross-ply layup $[0/90/0/90]_s$.
 a) 0° layer b) 90° layer c) 0° layer d) 90° layer

3.5. Stress field in angle-ply layup $[45/-45/45/-45]_s$

Friction factor between pin and plate in angle-ply layup was calculated. It was equal to 0.2461 and 0.2 in circumferential and thickness direction, respectively. Three different conditions were considered, according to friction direction. At first, friction was applied just in z direction. Second, it was just applied in θ direction and finally in both z and θ direction.

Based on the abovementioned conditions, radial stresses were plotted (Fig 14). It is seen that friction effect in θ direction is greater and more important than z direction. According to this, radial stress diagrams considering μ_z and without considering friction is so close in each layer and stress diagrams for μ_θ and $\mu_{\theta,z}$ have more resemblance.

It is also seen that in first layer (45°), maximum difference of MRSs for two conditions of $\mu = 0$ and μ_z is equal to 2.2% but no difference for μ_θ and $\mu_{\theta,z}$. This value for $\mu = 0$ and $\mu_{\theta,z}$ is 23%. The differences of MRSs are less at third layer (45°). For two conditions of $\mu = 0$ and μ_z , maximum stresses have 1.3% difference, no differences in MRSs for μ_θ and $\mu_{\theta,z}$ and for $\mu = 0$ and $\mu_{\theta,z}$, 20%.

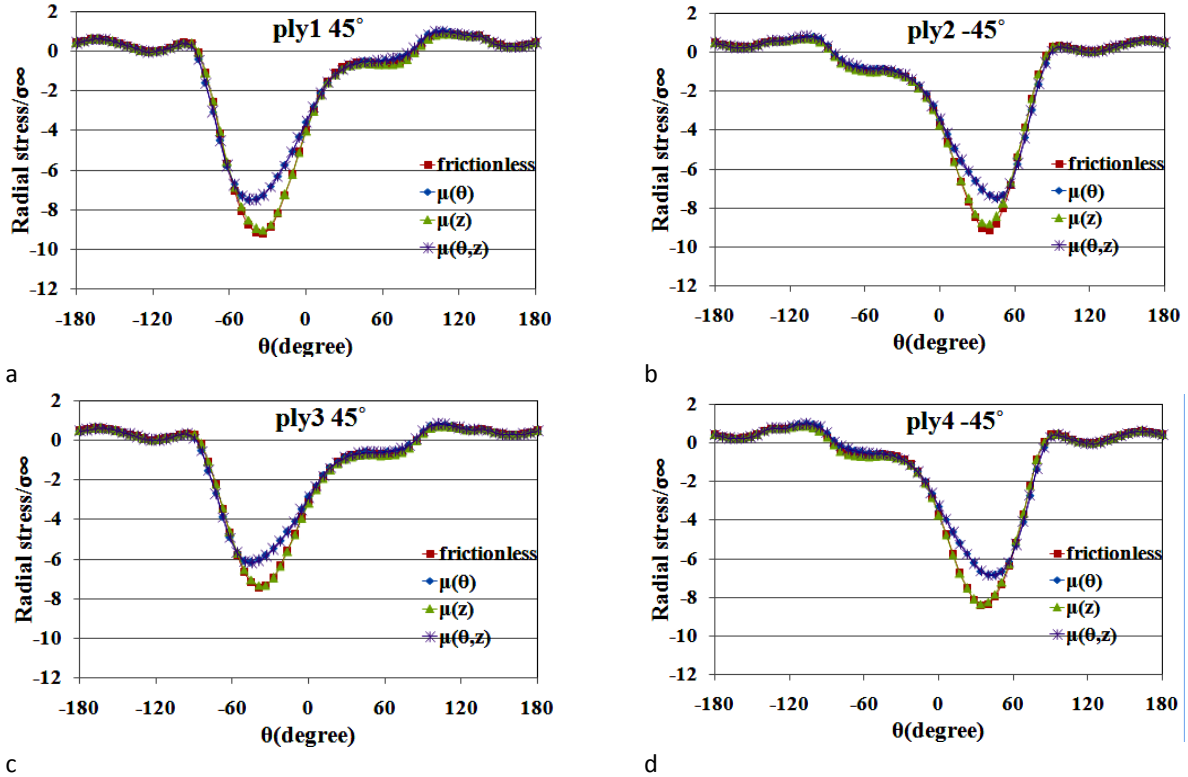
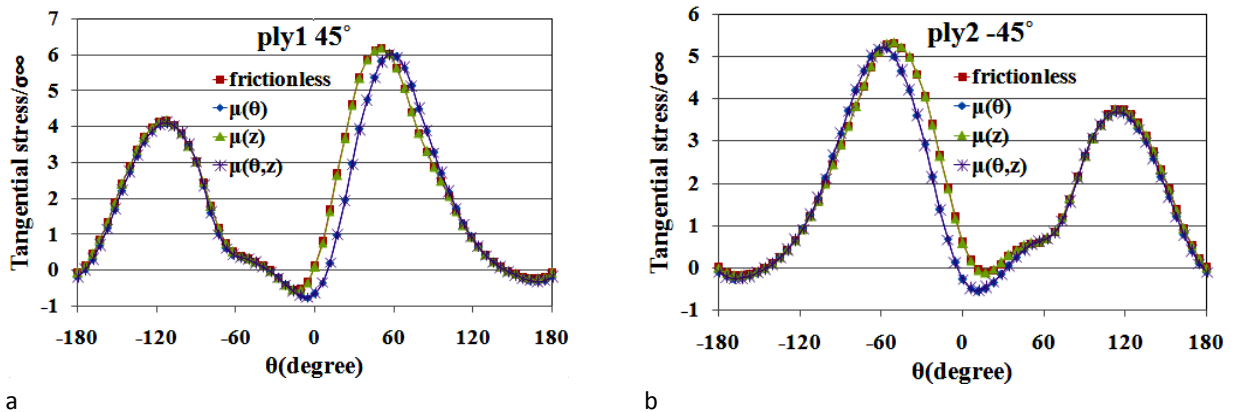


Fig. 14. Radial stresses comparison between layers in angle-ply layup $[45/-45/45/-45]$,
 a) 45° layer b) -45° layer c) 45° layer d) -45° layer

In second layer (-45°), the difference between MRSs for $\mu = 0$ and μ_z is equal to 4% and for μ_θ and $\mu_{\theta,z}$ no difference is observed. In this layer, the difference between maximum and minimum value of MRSs is 22%. Similarly in fourth layer (-45°), MRSs are same for $\mu = 0$ and μ_z ; but it is not alike for μ_θ and $\mu_{\theta,z}$, and the difference is 3%. Meanwhile difference between maximum and minimum value of MRSs equals 26%.

As expected, because of asymmetry of fibers and load direction, radial stress diagrams are not symmetric in all layers. Existence of friction affects location of maximum stress. Since friction effect in θ direction is more, location of maximum stresses for μ_θ and $\mu_{\theta,z}$ is similar. In the first and third layers, MRS appears at -33.8° and -40° for $\mu = 0$ and μ_z , respectively; and for μ_θ and $\mu_{\theta,z}$ at -40° and -45° .



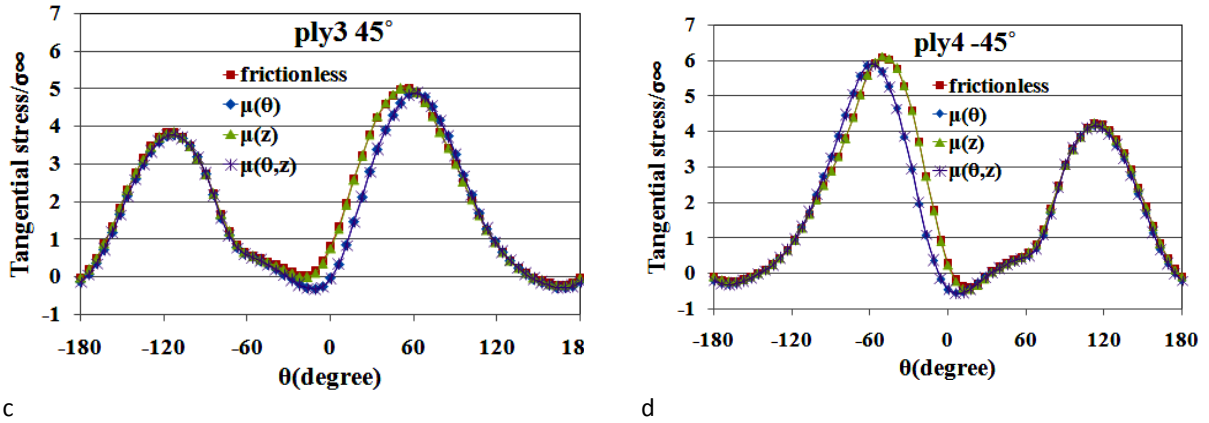


Fig. 15. Tangential stresses comparison between layers in angle-ply layout $[45/-45/45/-45]$.
 a) 45° layer b) -45° layer c) 45° layer d) -45° layer

In second and fourth layers, MRSs turn up at angles 33.8° and 40° for $\mu = 0$ and μ_z and at 40° and 45° for μ_θ and $\mu_{\theta,z}$, respectively. Friction in θ direction causes the location of maximum stress to rotate about 5° than $\mu = 0$ and μ_z . Tangential stresses are illustrated in Fig 15. According to this diagram, it is found that in first layer, for two conditions of $\mu = 0$ and μ_z and for μ_θ and $\mu_{\theta,z}$, there is no difference between MTSs. The difference between MTS for $\mu = 0$ and μ_θ is 3%. These values are the same for third layer (45°). For second layer (-45°), MTSs are 1% and 0% for conditions $\mu = 0$ and μ_z and μ_θ and $\mu_{\theta,z}$, respectively. Whilst the difference between conditions of $\mu = 0$ and μ_θ equals 2%. Fourth layer (-45°) is the same as second layer; except that the difference between conditions of $\mu = 0$ and μ_θ equals 3%.

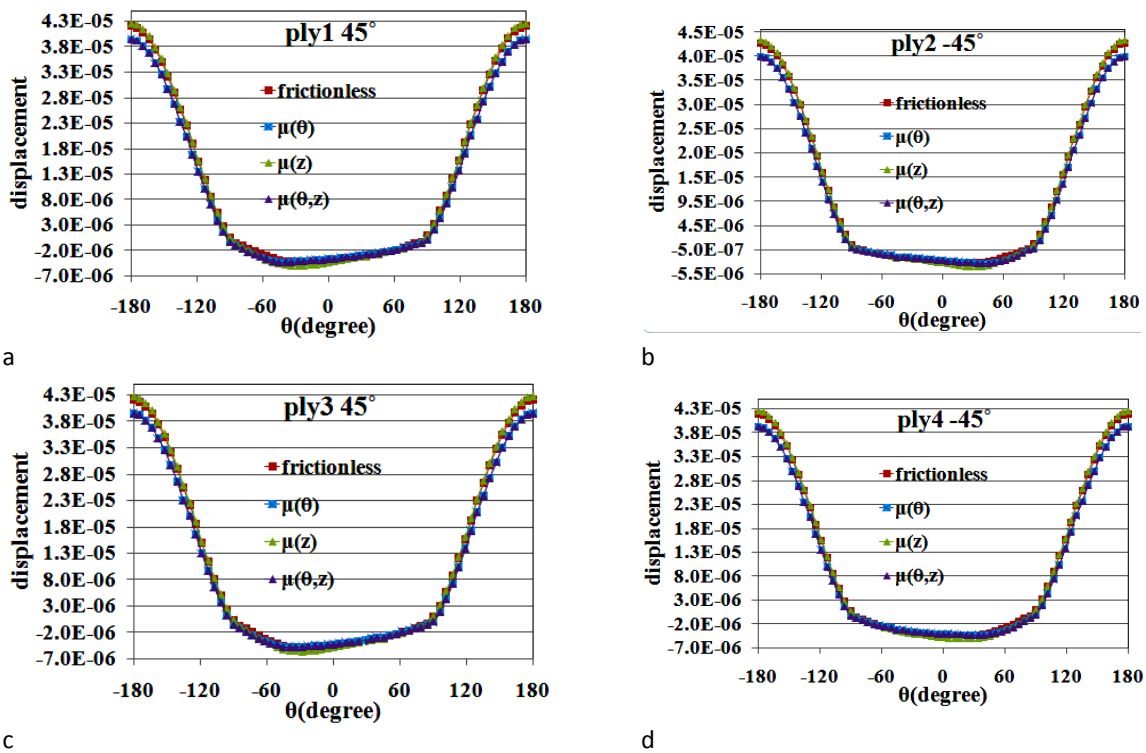


Fig. 16. Radial displacement comparison between layers in angle-ply layout $[45/-45/45/-45]$.

Since friction effect is greater in θ direction, location of MTS for μ_{θ} and $\mu_{\theta,z}$ is alike. In first and third layers MTSs occur for $\mu = 0$ and μ_z at 50° and 56°, respectively, and for μ_{θ} and $\mu_{\theta,z}$ at 56° and 61°, respectively.

In second and fourth layer, MTS for $\mu = 0$ and μ_z occur at -50° and for μ_{θ} and $\mu_{\theta,z}$ at -56°. Friction in θ direction causes that the location of MTS rotate about 5° toward conditions $\mu = 0$ and μ_z .

Radial displacement is depicted in Fig 16. As it is seen, on contact surface, friction has no significant effect on radial displacement.

4. Conclusion

In this paper, interlaminar stress in pin-composite plate joint and the effect of different stacking sequences on generated stresses between layers and their stress distribution were investigated. The effect of friction on stresses between layers in composite plates was also studied. The problem was modeled three dimensionally in ABAQUS software, considering interlaminar stresses, friction and different stacking sequence. As a result, some remarks are presented.

When sequence is symmetry, stress field is symmetry toward middle plate, too.

In each layer, MRSs (or MTSs) occur in areas that fibers direction is perpendicular (or tangent) to contact surface.

In 0° layer, that fibers and load directions are parallel; radial stress has its maximum value. But at 90° layer, Minimum value of radial stress occurs.

In quasi-isotropic layup $[0/45/90/-45]_s$, 45° and -45° layers cause asymmetry in radial and tangential stresses though in 0° and 90° layers, fibers direction is symmetric toward load direction. Whilst in cross-ply layers $[0/90/0/90]_s$, radial and tangential stresses are symmetric in all layers.

In cross-ply layup, stress fields are symmetry toward bearing and mid-plane.

In different layups, whatever angles are much closer, the difference between stresses is lesser; whereby in quasi-isotropic layup, the difference between stresses in 0° and 45° layers are lesser than 0° and 90° layers

Applying friction in different directions, it was seen that friction coefficient in θ direction was higher and more important than in other directions. As a result, radial and tangential stress diagrams for two condition of friction in z direction and without friction were so close and also stress diagrams for two condition of friction μ_{θ} and $\mu_{\theta,z}$ have more similarity.

In the first layer of cross-ply and angle-ply layup, friction coefficient $\mu_{\theta,z}$ causes a difference of 32% and 23% of MRSs, respectively, than condition of no friction. This difference is lesser in other layers.

In the second layer of cross-ply and angle-ply layup, friction coefficient μ_{θ} causes a difference of 50% and only 3% of MTSs, respectively, than condition of no friction. Again, this difference is lesser in other layers.

In cross-ply layup, friction has almost no effect on location of MRSs and MTSs.

In angle-ply layup, friction in θ direction forced the location of MRS and MTS to rotate about 5° toward the condition of no friction and μ_z .

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