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**Original article**

# Inference on stress-strength reliability of power distribution

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ABSTRACT

This paper deals with inference for the stress-strength reliability $R = P(Y < X)$ when $X$ and $Y$ are two independent power distribution. The problem of hypothesis testing and interval estimation of the reliability parameter in a stress-strength model is considered. Test and interval estimation procedures based on the generalized variable approach are given. Statistical properties of the generalized variable approach and an asymptotic method are evaluated by Monte Carlo simulation.



## 1. Introduction

In reliability contexts, inference about $R = P(Y < X)$, where $X$ and $Y$ are independent random variables, are a subject of interest. In mechanical reliability of a system, if $X$ is the strength of a component which is subject to stress $X$, then $R$ is a measure of system reliability. For a particular situation, consider $Y$ as the pressure of a chamber generated by ignition of a solid propellant and $X$ as the strength of the chamber. Then $R$ represents the probability of successful firing of the engine. In this context, $R$ can be considered as a measure of system performance and it is naturally arise in electrical and electronic systems. It may be mentioned that $R$ is of greater interest than just reliability since it provides a general measure of the difference between two populations and has applications in many area. For example, if $X$ is the response for a control group, and $Y$ refers to a treatment group, $R$ is a measure of the effect of the treatment. Also, it may be mentioned that $R$ equals the area under the receiver operating characteristic (ROC) curve for diagnostic test or biomarkers with continuous outcome. The ROC curve is widely used, in biological, medical and health service research, to evaluate the ability of diagnostic tests or bio-markers to distinguish between two groups of subjects, usually non-diseased and diseased subjects. For more details see Kotz et al. (2003). Many authors have studied the stress-strength parameter $R$. Among them,Ahmad et al.(1997), Awad et al.(1981), Baklizi and El-Masri (2004), Kundu and Gupta(2005, 2006), Adimari and Chiogna (2006), Baklizi (2008), Raqab et al. (2008) and Asgharzadeh et al. (2011).

The power distribution is one of the most widely used distributions in the reliability and survival studies. In this article, we want to develop inferential procedures about the reliability parameter $R = P(Y < X)$, where $X$ and $Y$ are independent power distribution random variables. The power distribution denoted by $\pi(b, \delta)$ has probability density function (PDF)

$$f(x;b,\delta)=\delta b^{-\delta} x^{\delta-1}, \qquad 0<x<b \quad , \quad x \in R, \quad (1)$$

and the cumulative distribution function (cdf)

$$F(x;b,\delta) = \left(\frac{x}{b}\right)^{\delta}, \qquad 0 < x < b, \quad (2)$$

Here $\delta$ is the shape parameter and b is the scale parameter.

This article is organized as follows. In section 2, we calculate parameter reliability in the power distribution. In section 3, we present the MLE of $R$. In section 4, we present the asymptotic results given in Kotz et al.(2003). In section 5, we explain first the method of constructing generalized variables for the parameters of a power distribution. Using these generalized variables, in section 6, we construct a generalized variable for the reliability parameter $R$ and outline the procedures for constracting confidence limits and hypothesis testing about $R$. The different proposed methods have been compared using Monte Carlo simulations and their results have been reported in Section 7. Finally, we conclude the paper in Section 8.

**2. Calculate the $R$**

Let $X \sim (b_1, \delta_1)$ and $Y \sim (b_2, \delta_2)$, where $\delta_1$, $\delta_2$ are known and $b_1$, $b_2$ are unknown. In order to evaluate the probability $P(Y < X)$, since

$$P(Y < X \mid X) = \begin{cases} (\frac{x}{b_2})^{\delta_2} & 0 < x < b_2 \\ 1 & x > b_2, \end{cases}$$

and

$$P(Y < X) = E(P(Y < X \mid X)),$$

then we obtain, if b $_1 >$ b $_2$

$$P(X > Y) = p_y\, p_{x|y}(X > Y \mid 0 < X < b_2) + p_y\, p_{x|y}(X > Y \mid b_2 < X < b_1)$$
$$= \int_0^{b_2} (\frac{x}{b_2})^{\delta_2} \delta_1 b_1^{-\delta_1} x^{\delta_1 - 1} dx + \int_{b_2}^{b_1} \delta_1 b_1^{-\delta_1} x^{\delta_1 - 1} dx = 1 - (\frac{b_1}{b_2})^{-\delta_1} \frac{\delta_2}{\delta_1 + \delta_2},$$

If $b_1 \le b_2$, Then $R$ is given by

$$p(X > Y) = E_y\, p_{x|y}(X > Y \mid Y)$$
$$= \int_0^{b_1} (\frac{x}{b_2})^{\delta_2} \delta_1 b_1^{-\delta_1} x^{\delta_1 - 1} dx = (\frac{b_1}{b_2})^{\delta_2} \frac{\delta_1}{\delta_1 + \delta_2},$$

Thus, the reliability parameter $R$ can be expressed as:

$$R = \left[ 1 - (\frac{b_1}{b_2})^{-\delta_1} \frac{\delta_2}{\delta_1 + \delta_2} \right] I(b_1 > b_2) + \left[ (\frac{b_1}{b_2})^{\delta_2} \frac{\delta_1}{\delta_1 + \delta_2} \right] I(b_1 \le b_2) \quad (3)$$

where I(.) is the indicator function.

If $b_1 = b_2$, then the reliability parameter $R$ simplifies to $\frac{\delta_1}{\delta_1 + \delta_2}$, and exact confidence limits for $R$ can be obtained using some conventional approaches. If $b_1 \ne b_2$, then the form for $R$, as shown in (3), is quite complex, and only large sample approach is available.

**3. Estimation of $R$**

If $X_1,..., X_n$ be a sample of observations from the power distribution with pdf in (1), the MLE's of b and $\delta$ are given by

$$l(b,\delta) = \delta^n b^{-n\delta} \left( \prod_{i=1}^{n} X_i \right)^{\delta - 1} I(x_{(n)} < b). \quad (4)$$

Upon using (4), we immediately have the MLE's of b and $\delta$ as follows:

$$\hat{b} = X_{(n)}, \quad \hat{\delta} = \frac{n}{\sum_{i=1}^{n} Ln \frac{X_{(n)}}{X_i}}, \quad (5)$$

where $X_{(n)}$ is the maximum of the $X_i$'s. It is known that $\hat{b}$ and $\hat{\delta}$ are independent with

$$2n\delta \ln(\frac{\hat{b}}{b}) : \chi^2_{(2)} \quad and \quad \frac{2n\delta}{\hat{\delta}} : \chi^2_{(2n)}. \quad (6)$$

Let $X$ be the power random variable with pdf $f(x; b_1, \delta_1)$ and $Y$ be the power random variable with pdf $f(y; b_1, \delta_2)$, where the pdf's are as defined in (1). Assume that $X$ and $Y$ are independent. Let $X_1,..., X_n$ be a sample of observations on $X$ and $Y_1,..., Y_m$ be a sample of observations on $Y$. Specifically, The MLE's are

$$\hat{b}_1 = X_{(n)}, \quad \hat{b}_2 = Y_{(m)}, \quad \hat{\delta}_1 = \frac{-n}{\sum_{i=1}^{n} \ln \frac{X_i}{X_{(n)}}} \quad and \quad \hat{\delta}_2 = \frac{-m}{\sum_{i=1}^{m} \ln \frac{Y_i}{Y_{(m)}}}, \quad (7)$$

The MLE of the reliability parameter $R$ can be obtained by replacing the parameters $b_1$, $b_2$, $\delta_1$ and $\delta_2$ in (3) by their MLE's. That is, the MLE of $R$ is given by

$$\hat{R} = \left[ 1 - (\frac{\hat{b}_1}{\hat{b}_2})^{-\hat{\delta}_1} \frac{\hat{\delta}_2}{\hat{\delta}_1 + \hat{\delta}_2} \right] I(\hat{b}_1 > \hat{b}_2) + \left[ (\frac{\hat{b}_1}{\hat{b}_2})^{\hat{\delta}_2} \frac{\hat{\delta}_1}{\hat{\delta}_1 + \hat{\delta}_2} \right] I(\hat{b}_1 \le \hat{b}_2) \quad (8)$$

**4. An asymptotic approach**

An asymptotic confidence interval for $R$ is given in kotz et al (2003). This confidence interval is based on an asymptotic distribution of the MLE of $R$. we shall now present an asymptotic mean squared error of $\hat{R}$ given in kotz et al (2003).We have

$$\frac{\partial R}{\partial \delta_1} = \begin{cases} (\frac{b_2}{b_1})^{\delta_1}\left[\frac{\delta_2}{(\delta_1+\delta_2)^2} - Ln(\frac{b_2}{b_1}\frac{\delta_2}{\delta_1+\delta_2})\right] & if \quad \hat{b}_1 > \hat{b}_2 \\ \frac{\delta_2}{(\delta_1+\delta_2)^2}\left(\frac{b_1}{b_2}\right)^{\delta_2} & if \quad \hat{b}_1 \le \hat{b}_2 . \end{cases} \quad (9)$$

Let $\lambda = \frac{n}{n+m}$ and define

$$D_j = \begin{cases} (\frac{\hat{b}_j}{\hat{b}_i})^2\left[\frac{\hat{\delta}_j}{(\hat{\delta}_i+\hat{\delta}_j)^2} - Ln\frac{\hat{b}_j}{\hat{b}_i}\frac{\hat{\delta}_j}{\hat{\delta}_i+\hat{\delta}_j}\right] & \text{if } \hat{b}_i > \hat{b}_j \\ \frac{\hat{\delta}_j}{(\hat{\delta}_i+\hat{\delta}_j)^2}[\frac{\hat{b}_i}{\hat{b}_j}]^{\hat{\delta}_j} & \text{if } \hat{b}_i \le \hat{b}_j \end{cases} \quad (10)$$

where i=1, if j=2 and i=2, if j=1. Using these terms, an estimate of asymptotic MSE of $\hat{R}$ given by

$$\hat{\delta}_{\hat{R}}^2 = d_1^2\frac{\hat{\delta}_1^2}{n} + d_2^2\frac{\hat{\delta}_2^2}{m}. \quad (11)$$

Using this estimator, for large n+m, we have the asymptotic distribution of $\frac{\sqrt{n+m}(\hat{R}-R)}{\hat{\delta}_{\hat{R}}}$ to be standard normal. A 100(1- $\alpha$ )% lower limit for $R$ based on the above asymptotic distribution is given

$$\hat{R} - z_{1-\alpha}\frac{\hat{\delta}_{\hat{R}}}{\sqrt{n+m}}, \quad (12)$$

Where $Z_p$ denotes the p-th quantile of the standard normal distribution.

## 5. Generalized variables for b and $\delta$

The reliability parameter in (3) is a function of both b's and $\delta$ 's. So we develop first generalized variables for b and $\delta$ for the one-sample case. Even though it is not our primary interest, knowing the results of the one-sample case will make it easier to understand the approach and results for the stress-strength reliability in section 6.

### 5.1. A Generalized variable for b

Let $\hat{b}_0$ and $\hat{\delta}_0$ be observed values of $\hat{b}$ and $\hat{\delta}$ respectively. Based on the above distributional results of the MLE's, a generalized pivot variable for b can be constructed as follows:

$$G_b = \exp\left(-2n\ln(\frac{\hat{b}}{b}) \times \frac{\hat{\delta}}{2n\delta} \times \frac{1}{\hat{\delta}_0 \ln\hat{b}_0}\right) = \exp\left(\frac{\chi^2_{(2)}}{\chi^2_{(2n)}} \times \frac{1}{\hat{\delta}_0 \ln\hat{b}_0}\right). \quad (13)$$

To get the last step, we used the distributional results in (6). The generalized test variable for testing hypothesis about b is given by

$$G_b^t = G_b - b = \exp\left( \frac{\chi^2_{(2)}}{\chi^2_{(2n)}} \times \frac{1}{\hat{\delta}_0 \ln \hat{b}_0} \right) - b. \quad (14)$$

In general, a generalized pivot variable should satisfy two properties. More details can be found in Weerahandi(1995b). Thus, we showed that $G_b$ is a bona fide generalized pivot variable for constructing confidence limits for b, and $G_b^t$ is a valid generalized test variable for hypothesis testing about b. For example, the 100$\alpha$ -th percentile of $G_b$, that is:

$$G_b = \exp\left( \frac{\frac{\chi^2_{(2)}}{2} \times 2}{\frac{\chi^2_{(2n)}}{2n} \times 2n} \times \frac{1}{\hat{\delta}_0 \ln \hat{b}_0} \right) = \exp\left( \frac{2}{2n} F_{2,2n,1-\alpha} \times \frac{1}{\hat{\delta}_0 \ln \hat{b}_0} \right), \quad (15)$$

where $F_{m,n,p}$ denotes the 100p-th percentile of the F distribution with degrees of freedoms m and n, is a 100(1-$\alpha$)% lower confidence limit for b. If one is interested in testing

$$H_0 : b \le b_0 \; vs. H_1 : b > b_0$$

Then, noting that $G_b^t$ is stochastically decreasing in b, the generalized p-value is given by

$$p\left( \sup_{A_0} G_b^t < 0 \right) = p\left( G_b^t < 0 \mid b = b_0 \right)$$
$$= p\left( G_b < b_0 \right).$$

Using (13), and after some simplification, we see that the above p-value can be expressed as

$$p\left( \frac{2}{2n} F_{2,2n} < \frac{\ln b_0}{\hat{\delta}_0 \ln \hat{b}_0} \right).$$

The test or interval estimation of b based on our generalized variable approach are the same as the usual exact ones (see lawless 1982, p.128).

### 5.2. A Generalized Variable for $\delta$

A generalized variable for $\delta$ is given by

$$G_\delta = \frac{2n\delta}{\hat{\delta}} \times \frac{\hat{\delta}_0}{2n} = \frac{\hat{\delta}_0}{2n} \chi^2_{(2n)}. \quad (16)$$

And the generalized test variable based on $G_\delta$ is given by

$$G_\delta^t = \frac{\hat{\delta}_0}{2n} \chi^2_{(2n)} - \delta. \quad (17)$$

It is easy to see that the generalized pivot variable and the generalized test variable and the generalized test variable satisfy the properties given earlier.

**6. Generalized confidence limits for $R$**

The generalized variable for $R$ can be obtained by replacing the parameters by their generalized variables. The reliability parameter $R$ simplifies to $\frac{b_1}{b_1+b_2}$ when $b_1=b_2$. Denoting the resulting generalized variable by $G_R$, we have

$$G_R=\left[1-\left(\frac{G_{b_1}}{G_{b_2}}\right)^{-G_{\delta_1}}\frac{G_{\delta_2}}{G_{\delta_1}+G_{\delta_2}}\right]I\left(G_{b_1}>G_{b_2}\right)+\left[\left(\frac{G_{b_1}}{G_{b_2}}\right)^{G_{\delta_2}}\frac{G_{\delta_1}}{G_{\delta_1}+G_{\delta_2}}\right]I\left(G_{b_1}\le G_{b_2}\right), \tag{18}$$

where

$$G_{b_i}=e^{\frac{Q_i}{W_i}\times\frac{1}{\hat{\delta}_{io}\ln\hat{b}_{io}}},G_{\delta_1}=\frac{\hat{\delta}_{10}}{2n}w_1,G_{\delta_2}=\frac{\hat{\delta}_{20}}{2m}w_2,i=1,2.$$

here $\left(\hat{b}_{io},\hat{\delta}_{io}\right)$ is an observed value of $\left(\hat{b}_i,\hat{\delta}_i\right), i=1,2$, and $Q_1, Q_2, W_1$ and $W_2$ are independent random variables with $Q_i:\ \chi^2_{(2)}$ and $w_1:\ \chi^2_{(2n)}, w_2:\ \chi^2_{(2m)}, i=1,2.$ The generalized test variable for $R$ is given by $G_R^t=G_R-R$.

It is easy to check that the generalized pivot variable $G_R$ satisfies the two properties given in section. Monte Carlo method given in algorithm 1, can be used to find confidence limits for $R$ or to find the generalized p-value for hypothesis testing about $R$.

**Algorithm 1:**

1. For a given data set, compute the MLEs $\hat{b}_{10},\hat{b}_{20},\hat{\delta}_{10},\hat{\delta}_{20}$ using the formulas in (3) for $i=1,m.$
2. Generate $Q_1:\ \chi^2_2, Q_2:\ \chi^2_{(2)}, W_1:\ \chi^2_{(2n)}, W_2:\ \chi^2_{2m}.$
3. Compute $G_{b_1},G_{b_2},G_{\delta_1},G_{\delta_2}$ and $G_R$.

The 100 $\alpha$ -th percentile of the generated $G_R$'s is a 1- $\alpha$ lower limit for the reliability parameter R. If we are interested in testing

$$H_0:R\le R_0\ vs.\ H_1:R>R_0,$$

Where $R_0$ is a specified value, then the generalized p-value is the proportion of the $G_R$'s that are less than $R_0$.

**7. Simulation study**

In this section we compare the performance of the two methods through a simulation study. We assume that $b_2=2.5,\delta_2=1$ to compute the coverage probabilities. We used different sets of parameter values $(\delta_1=1,2,3,5)$ and $(b_1=1,3,3.5,4,4.5)$ mainly to compare the coverage probabilities and expected lengths of 95% lower confidence limits for R. The simulation is carried out as follows. For a given $\left(n,b_1,\delta_1,m\right)$, we first generated 2000 $(\hat{b}_{10},\delta_{10},\hat{b}_{20},\delta_{20})$'s using the distributional results in (5). For each simulated set

$(\hat{b}_{10}, \delta_{10}, \hat{b}_{20}, \delta_{20})$, we used algorithm1with $m = 3000$ to find the 95% lower limit for $R$. The proportion of the 2000 lower limits that are below the value of $R$ is a Monte Carlo estimate of the coverage probability. The coverage probabilities of the asymptotic limit in (6) were estimated using simulation consisting of 10000 runs.

In Tables 1and 2, we present coverage probabilities of asymptotic limits and generalized confidence limits for sample n=m=50 and n=m=100. We chose large sample size because the asymptotic limits are valid only for large samples. In Table 3, we give the coverage probabilities of the generalized limits for small samples. Furthermore, to understand the closeness of the lower confidence limits to the value of the reliability parameter, we present estimates of the expectation of the lower limits and the value $R$ for each parameter and sample size configurations. Correspond to each different sets of parameter values, the first row represents the Coverage probabilities of generalized limit, and the corresponding expected lengths are reported within brackets. Similarly, the second line represents the results corresponding to asymptotic limit of $R$. Some of the points are quite clear from Tables 1,2 and 3.

• We observe from Table 2 that, even for large samples, the coverage probabilities of the asymptotic approach are in general smaller than the nominal level 0.95, and for fixed $b_1$, as $\delta_1$ increases in most of cases the coverage probabilities of the asymptotic limits increase.

• The coverage probabilities of the generalized confidence limits are in general either close to or slightly more than the nominal level 0.95. Comparison of values for n=10 and m=15(in Table 3) and those for n=m=50 (in Table 1) suggests that the coverage probabilities approach nominal level as sample sizes increase. Thus, we see that the generalized inference is in general conservative, and its accuracy increases as the sample sizes increase.

• Comparison between the estimates of the expectation of the lower limits and the values of $R$ indicates that the lower limits are expected to be fairly close to $R$ even though the generalized estimation procedures is slightly conservative. Furthermore, for fixed confidence level, the lower limits tend to increase as the sample sizes increase. For example, when n=10 and m=15 and $b_1 = 3, \delta_1 = 2, R = 0.80$ and the lower limit is 0.62 (see Table 3); at the same parameter configuration, the lower limit is 0.73 when n=m= 50 (see Table 1) and is 0.75 when n=m= 100 (see Table 2). Thus, the lower confidence limit is expected to increase with increasing sample sizes, which is a desirable property.

• The size properties of the generalized test can be understood from the above coverage properties. In particular, the sizes of the test should be close to or less than the nominal level, and they are expected to be close to the nominal level for large samples.

**Table 1**

Coverage probabilities (CP) and expected lengths (EL) of 95% lower confidence limits for R when n=m=50 and $b_2 = 2.5, \delta_2 = 1$.

| | $\delta_1 = 1$ | | $\delta_1 = 2$ | | $\delta_1 = 3$ | | $\delta_1 = 5$ | |
|---|---|---|---|---|---|---|---|---|
| $b_1$ | R | CP(EL) | R | CP(EL) | R | CP(EL) | R | CP(EL) |
| 1 | 0.11 | 0.92(0.07) | 0.31 | 0.93(0.24) | 0.45 | 0.95(0.38) | 0.55 | 0.95(0.47) |
| | | 0.77(0.06) | | 0.91(0.18) | | 0.94(0.29) | | 0.95(0.39) |
| 3 | 0.70 | 0.93(0.61) | 0.80 | 0.92(0.73) | 0.85 | 0.93(0.78) | 0.88 | 0.92(0.82) |
| | | 0.91(0.50) | | 0.95(0.69) | | 0.96(0.78) | | 0.96(0.84) |
| 3.5 | 0.82 | 0.95(0.74) | 0.88 | 0.92(0.82) | 0.91 | 0.92(0.86) | 0.93 | 0.94(0.88) |
| | | 0.90(0.56) | | 0.93(0.77) | | 0.95(0.86) | | 0.95(0.91) |
| 4 | 0.89 | 0.95(0.82) | 0.93 | 0.92(0.88) | 0.94 | 0.93(0.90) | 0.96 | 0.93(0.92) |
| | | 0.91(0.60) | | 0.92(0.81) | | 0.94(0.89) | | 0.94(0.94) |
| 4.5 | 0.93 | 0.93(0.88) | 0.95 | 0.95(0.91) | 0.97 | 0.92(0.94) | 0.96 | 0.95(0.95) |
| | | 0.92(0.63) | | 0.94(0.83) | | 0.94(0.91) | | 0.95(0.95) |

**Table 2**

Coverage probabilities (CP) and expected lengths (EL) of 95% lower confidence limits for R when n=m=100 and $b_2 = 2.5, \delta_2 = 1$.

| | $\delta_1 = 1$ | | $\delta_1 = 2$ | | $\delta_1 = 3$ | | $\delta_1 = 5$ | |
|---|---|---|---|---|---|---|---|---|
| $b_1$ | R | CP(EL) | R | CP(EL) | R | CP(EL) | R | CP(EL) |
| 1 | 0.11 | 0.95(0.08)<br>0.82(0.07) | 0.31 | 0.91(0.26)<br>0.93(0.21) | 0.45 | 0.95(0.40)<br>0.94(0.31) | 0.55 | 0.95(0.49)<br>0.95(0.40) |
| 3 | 0.70 | 0.94(0.63)<br>0.92(0.53) | 0.80 | 0.95(0.75)<br>0.94(0.72) | 0.85 | 0.95(0. 80)<br>0.95(0.80) | 0.88 | 0.93(0.84)<br>0.95(0.86) |
| 3.5 | 0.82 | 0.93(0.76)<br>0.92(0.59) | 0.88 | 0.95(0.83)<br>0.94(0.79) | 0.91 | 0.95(0.87)<br>0.95(0.87) | 0.93 | 0.93(0.90)<br>0.95(0.92) |
| 4 | 0.89 | 0.95(0.85)<br>0.93(0.63) | 0.93 | 0.94(0.89)<br>0.95(0.83) | 0.94 | 0.93(0.92)<br>0.95(0.91) | 0.96 | 0.93(0.93)<br>0.96(0.95) |
| 4.5 | 0.93 | 0.95(0.90)<br>0.94(0.66) | 0.95 | 0.93(0.93)<br>0.94(0.85) | 0.97 | 0.94(0.95)<br>0.95(0.93) | 0.97 | 0.94(0.96)<br>0.94(0.96) |

**Table 3**

Coverage probabilities (CP) and expected lengths (EL) of 95% lower confidence limits for R when n=10, m=15 and $b_2 = 2.5, \delta_2 = 1$.

| | $\delta_1 = 1$ | | $\delta_1 = 2$ | | $\delta_1 = 3$ | | $\delta_1 = 5$ | |
|---|---|---|---|---|---|---|---|---|
| $b_1$ | R | CP(EL) | R | CP(EL) | R | CP(EL) | R | CP(EL) |
| 1 | 0.11 | 0.93(0.04) | 0.31 | 0.94(0.17) | 0.45 | 0.94(0.28) | 0.55 | 0.96(0.36) |
| 3 | 0.70 | 0.94(0.52 ) | 0.80 | 0.95(0.62) | 0.85 | 0.97(0.68) | 0.88 | 0.97(0.71) |
| 3.5 | 0.82 | 0.95(0.67) | 0.88 | 0.96(0.73) | 0.91 | 0.96(0.76) | 0.93 | 0.95(0.78) |
| 4 | 0.89 | 0.92(0.53) | 0.93 | 0.94(0.72) | 0.94 | 0.95(0.80) | 0.96 | 0.95(0.86) |
| 4.5 | 0.93 | 0.93(0.76) | 0.95 | 0.95(0.83) | 0.97 | 0.95(0.87) | 0.97 | 0.93(0.90) |

## 8. Conclusions

In this paper we have considered the classical inference procedure for the stress-strength parameter of power distribution. Test and interval estimation procedures based on the generalized variable approach are given. Statistical properties of the generalized variable approach and an asymptotic method are evaluated by Monte Carlo simulation. Simulation studies show that the proposed generalized variable approach is satisfactory for practical applications while the asymptotic approach is not satisfactory even for large samples.

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