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Strong interaction of hadrons in quark cluster model

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A B S T R A C T

The theoretical information on the hadrons interactions according to the basis investigation of the multiple scattering process theory is described. As we know multi-particle reactions on the hadrons targets are attracting a great attention nowadays. To survey strong interaction of jet particles with quarks that are inside hadrons (*Baryons,mesons, exotic baryons(Penta-quarks), exotic mesons(Tetra-quarks)*, we can use the estimate called high energy approximation (*Eikonal or Glauber approximation)* theory that known very well in nuclear physics. This estimate describes collision and interactions of jet particles with quarks and scattering from multi-focus hadrons like diffraction phenomenon in optics. Glauber multiple scattering process theory may apply in analyzing elastic and inelastic collision of hadrons in a range of high energy levels. In elastic collision, scattering amplitude is equal to total ranges of multiple collisions inside the hadrons. It's possible to express Glauber multiple scattering factor in a form of mathematic series. So that each elements shows the number of occurred scattering inside the hadrons. Determination of scattering amplitude by the high energy approximation depends on elected primary coming wave function of the shot particle and function of out coming wave from the target nucleus. Therefore it's not so hard to determine scattering amplitude. *The main purpose* of this paper is to show how to determine mathematical formula for differential cross section of jet particles in high energy levels with a hadrons in cluster model (*qq, qq*) (*Quarkonium-Quarkonium cluster)*.

1. Introduction

Hadron interactions are strong and, in principle, should be described byquantum chromo-dynamics. However, theoretical data and theories show that their main features can be originated from the nuclear interaction theory in high energy levels. In fact, our approach to high energy hadronic processes at present is at best still in its infancy. As has been learned from experiment data in nuclear interactions, strong interactions of high energy particles give rise to inelastic and elastic processes. Mostly quarkonium systems are produced in inelastic processes, which are the most probable ones, up than 2/3 of all processes at high energies. Most created particles or clusters have comparatively small transverse momenta and at the same time, the particles or cluster do not change their nature and scatter elastically, declining at some angle from their initial trajectories. In the light diffraction phenomenon in optics phenomenon we observe that wavelength of particle, is much lower than the size of the collide particle and hadronic target absorbs all incident waves to the hadron, and it can be considered as a complete dark body. Such a collision has a particular position in the physic of elastic and strong collision. Collision of particles in high energy with very small momentum compared with size of the hadronic target, will be a complete absorbed collision and the hadronic target can be considered as fully or semi dark body (Landsberg, 1979).Therefore collision between particles in high energy has light diffraction properties and through expands of diffraction phenomenon we will can survey high energy hadrons collision with hadrons in high energies, and express nuclear diffraction phenomenon applying light diffraction phenomenon. In nuclear diffraction phenomena, observes collision of incident particles with hadronic target applying separate collision of those particles inside the jet hadrons with each particle inside the hadronic target, then one surveys properties of the collision. Total amplitude of collision is the sum of the amplitude of incident particles inside the jet hadronic particle with each particle inside the hadronic target, which has a direct relation with the hadronic shape (which relates to quark structure and bound states formation of quarks: $\bar{q}q, qq\bar{q}, qqqq, q\bar{q}qq\bar{q},$ and etc.). For the dark body in diffraction phenomenon scattering elements matrix (scattering amplitude) is defined as following:

$$f(\vec{k}',\vec{k}) = -\frac{m}{2\pi\hbar^2}\langle k'|\hat{t}|k\rangle, \quad (1$$

where $\hat{t}$, - is converting factor from primary to secondary status. In small scattering angles where the angle between $\vec{k}',\vec{k}$ vectors is small, incident amplitude in high energies will be maximum, when there is not such a maximum, we can conclude that changes in momentum is small, in other word $\vec{k}' \cong \vec{k}$. Momentum is almost equal to primary momentum if considering very trivial changes of momentum in high energy physic. Therefore collision procedure will be explained and surveyed only in middle position that named after high energy approximation which also known as Eikonal or Glauber approximation. In hadronic physic, we have hadrons, which constitute by quarks (two quarkonium bound states system), so this model lets us theoretically describe structure of jet hadron and di-quarkoniumtargetsinteraction in high energy levels. In the cluster model, there is particular condition which particles in the hadron, composted bound states clusters that can be demonstrate as free bound states and they are more semi stable, in this state clusters demonstrate special properties, and its hadronic target will no longer be separated. Two quarkoniumbound state are one of semi-stable formation of hadronic cluster, which are very useful to explain collision in hadrons such as exotic baryons and exotic mesons with hadronic jets. Surveying multi-particle and cluster systems in collision physic with high energy is very important, so that nowadays there is the oretically calculating model which is applied to describe and calculate differential cross section of these reactions. Collision between jet hadrons and hadronic target in cluster model of quarkonium and quark were studies in this articles and transformation to cylindrical coordinate model and also equations about Jacobean particle coordination system transformation has been explained completely, so to have better understanding of equations of this article we suggest referring to old articles. In this article we will have look on collision procedure and cluster effects in scattering amplitude of jet hadrons from the smallest and unstable three quarks hadronsquarkonium-quark cluster (Glauber, 1967b; Roger, 1982; Charles, 1975).

2. Theoretical frameworks of cluster model

The interaction of hadrons with hadronic target is the main subjects in theoretical and experimental high energy physics that actively studied since 1993. At given high energy levels, the basic hadronic interaction is completely elastic which is under discussion here. The elastic interaction of jet hadrons with hadronic target is of considerable theoretical interest for an important reason. The main purpose of such investigations is to understand the mean field encountered by the inside particles of the jet hadron while traversing the target two quarkonium, system. This field is usually described in terms of either multiple scattering processes which depend upon quarks charge/color distributions or the complex optical model potential (Leonard, 1968; Frauenfelder and Henley, 1977). We will restrict ourselves here to the first one where the projectile undergoes multiple scatterings during its passage through the hadronic target in cluster model. These calculations are carried out for the elastic collisions, hadron-hadron at high energy levels. The full series calculations of differential cross with multiple interactions are better in describing the scattering data than using the single scattering. We have studied the scattering effects in hadron-hadron scattering. A single channel cluster model is used to calculate the wave function for the ground states of hadronic target. The multiple scattering theories (it is the well-known Glauber's multiple scattering theory (GMST) have been used to describe the jet hadron elastic scatterings from quarkonium-quark cluster model which is under discussion here. The theory is based on high energy Eikonal approximation, in which the interacting particles are almost frozen in their instantaneous positions during the passage of the projectile through the target and the trajectory is nearly straight forward that means momentum of indicate particles proximally equivalent with momentum of scattered particles. However, because the Glauber theory is principally derived for the higher energy and the small-angle situations, the reliability of its results may be questioned in the case of low energy and large angle. The GMST has the great advantage of leading to straightforward calculations of the elastic jet hadron-hadronic target in cluster model (jet hadron, quarkonium and quark target) scattering cross-sections from knowledge of free hadron-hadronic scattering amplitude and quark densities. The preliminary applications of GMST were found to have great successes in reproducing the hadron-hadronic scattering data. The confidence in this theory encouraged the extension of its application to hadron-hadronic target collisions. Correlations within the targets are of fundamental theoretical interest but unfortunately, they are difficult to study experimentally. In hadron- two quarkonium targets in cluster model scattering, they manifest themselves through a change of the effective hadron-hadronic cross section via the so-called in-medium hadron-hadronic amplitude. With regard that scattered targets resulted from collision in long distances from the center of collision are moving freely in the bound states description, and their relative motion energy always would remain positive. Therefore, considering Eikonal approximation condition in elastic collision, energy spectrum of jet/targets collision (Charles, 1975))having m as mass, in $\mathrm{V}(\vec{\mathrm{r}})$ potential filed and using Green function to study the bound states collision, which will be carried out on the research basis asymptotical Behaviour of two quarkonium targets in gauge field. In this case, loop function of two quarkonium targets which created from two scalar particles with different masses $\mathrm{m}_1$and m_2, with average on external statistical field is considered and the Green function is used. The Green function $\mathrm{G}\left(\vec{r}\ ,\ \vec{r}'\middle|\mathrm{A}\right)$ for scalar particles inside quarkonium in an external field is determined from the equation:

$$(\nabla^2 + k^2)G(\vec{r}-\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') \Rightarrow \tag{1}$$

$$\left[\left(i\frac{\partial}{\partial x_a} + \frac{g}{c\hbar}A_a(x)\right)^2 + \frac{c^2m^2}{\hbar^2}\right]G(\vec{r}\ ,\ \vec{r}\,'|A) \tag{2}$$

where m is the mass of a scalar particles inside hadronic targets, and g is the coupling constant of interaction.

As we know the scattering wave function has the form ($\varphi_0 = e^{ik\vec{z}}$ -Incident wave function, $f(\theta,\varphi)$ -Scattered wave function):

$$\Psi(\vec{r}) = \varphi_0 + f(\theta,\varphi)\frac{e^{i\vec{k}\vec{r}}}{\vec{r}} \quad (3$$

$$f(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d^3\vec{r}\,' \cdot \frac{e^{i\vec{k}|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}V(\vec{r}\,')\phi(\vec{r}\,') \quad (4)$$

We used cylindering coordinate, gave $\hbar = c = 1$ and ($oz \parallel \vec{k}$)by substitution to Eq.(3), then we can write:

$$\Psi(\vec{r}) = e^{i\vec{k}\vec{r}} - \frac{im}{k}\iint d\vec{z}\,' \cdot V(\vec{\rho},\vec{z}\,') \quad (5)$$

in this theory, the elastic scattering amplitude between a jet hadron and two quarkonium targets is:

$$f(\vec{k}\,',\vec{k}) = -\frac{m}{2\pi\hbar^2}\int d\vec{r}\,\frac{e^{i\vec{k}\vec{r}-im}}{k}\cdot e^{-i\vec{k}'\vec{r}}\cdot V(\vec{r})\int d\vec{z}\,'V(\vec{\rho},\vec{z}\,') \quad (6)$$

where k, -is scattering momentum and k'- is the incident momentum of jet hadron. Inserting transferred momentum q=k-k'in equation (6):

$$f(\vec{q}) = \frac{-m}{2\pi}\int d\vec{\rho}.d\vec{z}\cdot e^{-i\vec{q}\vec{r}}V(\vec{\rho},\vec{z})\cdot e^{\frac{-m}{k}\int d\vec{z}\,'.V(\vec{\rho},\vec{z}\,')} =$$

$$= \frac{-m}{2\pi}\int d\vec{z}\cdot e^{-i\vec{q}\vec{r}}V(\vec{\rho},\vec{z})\cdot e^{\frac{-im}{k}\int d\vec{z}\,'.V(\vec{\rho},\vec{z}\,')} \quad (7)$$

now use

$$\Psi(\vec{r}) = Re^{i\vec{k}\vec{r}},\; R = e^{\frac{-im}{k}\int d\vec{z}\,'.V(\vec{\rho},\vec{z}\,')}$$

where R is function, inserting $2ik\frac{\partial R}{\partial z} = 2mVR$ into eq. above, we obtain:

$$f(\vec{q}) = \frac{-ik}{2\pi}\int d\vec{\rho}\cdot e^{i\vec{q}\vec{\rho}}\left[1 - \frac{im}{k}\int V(\vec{\rho},\vec{z}\,')d\vec{z}\,'\right] \quad (8)$$

and we determinate:

$$f(\vec{q}) = \frac{ik}{2\pi}\int d\vec{\rho}\cdot e^{i\vec{q}\vec{\rho}}\left[1 - e^{-\frac{m}{k}\int d\vec{z}\,'.V(\vec{\rho},\vec{z}\,')}\right] \quad (9)$$

In describing scattering amplitude and properties of the collision system, profile function plays an important role and incident wave will has phase δl as result of each collision with nucleon and finally scattered wave (out coming) will have scattering phase equal to $\delta(\vec{\rho}),$ which can be stated as the result of sum of scattering phases with single targets $\delta_l = (\vec{\rho},\vec{\rho}_l),$ $\vec{\rho}$- is the position vector of hadronic target. Surface vectors $\vec{\rho},\vec{\rho}_l -$ are equal to image of radius vector of incident targets $\vec{r}$ and scattered jet hadrons $\vec{r}'$ in a plate perpendicular to primary momentum of $\vec{k}$ of incident particle.Accordingly, the modified optical phase-shift function $\delta(\vec{\rho})$, can be written sum phase-shift function for hadron- hadronic targets scattering, which is equal to (Anderson, 1977):

$$2\delta(\vec{\rho}) = -\frac{m}{\vec{k}} \int d\vec{z}' \cdot V(\vec{\rho}, \vec{z}'). \quad (10)$$

Applying functions (9, 10) and performing some sort of replacements, we will have the following relations:

$$f(\vec{q}) = \frac{i\vec{k}}{2\pi} \int d\vec{\rho} \cdot e^{i\vec{q}\vec{\rho}} \left[1 - e^{2\delta(\vec{\rho})}\right], \quad (11)$$

or

$$\omega(\vec{\rho} - \vec{\rho}_l) = \frac{1}{2\pi\vec{k}} \int d\vec{q} \cdot e^{i\vec{q}(\vec{\rho} - \vec{\rho}_l)} f(\vec{q}), \quad (12)$$

where $\omega(\vec{\rho}) = 1 - e^{2\delta(\vec{\rho})}$ -is total profile function.

Profile function of whole of the hadronic target is always equal to total of profile function of particles, and this is known as the first theoretical approximation of particles diffraction (Wieder, 1973). We define the scattering wave function of the ground state hadronic targets in hadron-hadronic elastic scattering as follow:

$$f(\vec{q}) = \frac{i\vec{k}}{2\pi} \int d\vec{\rho} \cdot e^{i\vec{q}\vec{\rho}} \omega(\vec{\rho}) \cdot \quad (13)$$

The survey of cluster models used at the same time for the description of the properties of exotic baryons and exotic mesons was carried out. It was shown that two quarkonium cluster model describes different characteristics of exotic hadrons. On the basis of the multiple diffraction scattering theory and the two quarkonium cluster model with dispersion the approach for description of the observables for elastic scattering of hadrons by two quarkonium cluster was considered. The calculated on the basis of this approach differential cross sections and polarization observables for elastic scattering of hadronsby quarkonium cluster were presented. These results testified that two quarkonium cluster model allows us to explain different experimental data for the scattering of intermediate energy particles by hadrons. Hadron in two quarkonium cluster model look like(Glauber, 1967a; Davidov, 1973; Sitenko, 1971; Zhusupov, 2000; Jhanshir, 2009; Jahanshir, 1999):

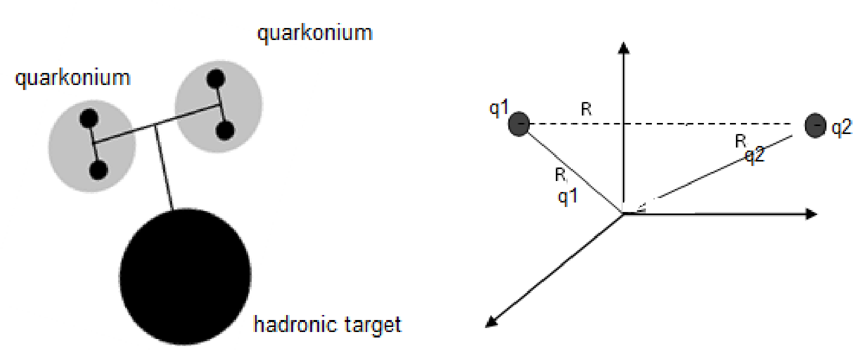

Fig. 1. Hadronic target in two quarkonium cluster model(left), Coordinate of hadronic targets(right).

The coordinates of the center of mass in Jacobean coordinate, coordinate of quarkonium targets are:

$$\vec{a} = \vec{r}_1 - \vec{r}_2, \quad \vec{R}_{q2} = \frac{\vec{r}_1 + \vec{r}_2}{2}, \quad \vec{R}_{q1} = \frac{\vec{r}_3 + \vec{r}_4}{2}$$

$$\vec{r}_1 = R_{q2} + \frac{\vec{a}}{2}, \quad \vec{r}_2 = \vec{R}_{q2} - \frac{\vec{a}}{2}$$

$\vec{R} = \vec{R}_{q1} - \vec{R}_{q2} -$ Coordinate of relative movement.

In calculation of final amplitude of hadron-hadronic scattering in cluster model, we should move from single particle coordination system of quarks to Jacobean coordination system (Jacobean Coordinate is used to represent) and also according to cluster model, jet hadron is scattering with two quarkonium targets that means it is not scatter with all quarks in hadronic target. The quantum numbers of hadronic target in the ground state should be given $(LM_L SM_S \mid JM)$. After performing a series of replacements and applying dimensional calculation, we will find new coordination of clusters and relative coordination of particle motion and coordination of center of total mass of the targets and replacing new coordination, jet hadron scattering amplitude of two quarkonium and each couple particle will be equal to following equations (For all details see[13,14]):

$$\Psi_H = (LM_L SM_S \mid JM)\phi_{q1}\phi_{q2}\phi_{q1q2}\chi_{S,M_S} \quad (15)$$

$$f(\vec{q}) = f_1(\Omega_{q1}) + f_2(\Omega_{q2}) - f_3(\Omega_{q1q2}) \quad (16)$$

where

$$\Omega_{q1} = \omega(\vec{\rho} - \vec{\rho}_{q1}) = \frac{1}{2i\pi\vec{k}}\int d\vec{q} \cdot e^{-i\vec{q}(\vec{\rho}-\vec{\rho}_{q1})} f(\vec{q}) \quad (17)$$

$$\Omega_{q2} = \omega(\vec{\rho} - \vec{\rho}_{q2}) = \frac{1}{2i\pi\vec{k}}\int d\vec{q} \cdot e^{-i\vec{q}(\vec{\rho}-\vec{\rho}q2)} f(\vec{q}) \quad (18)$$

$$\Omega_{q1q2} = \Omega_{q1}\Omega_{q2} = \omega(\vec{\rho} - \vec{\rho}_{q1q2\tau}) = \frac{1}{2i\pi\vec{k}}\int d\vec{q} \cdot e^{-i\vec{q}(\vec{\rho}-\vec{\rho}q1q2)} f(\vec{q}) \quad (19)$$

here χ_{S,M_S} - spins function of particles; $\phi_{q1}, \phi_{q2}, \phi_{q1q2}$ -Wave function of each cluster and have function of relative movement of both clusters; $f_1(\Omega_{q1}),\ f_2(\Omega_{q2}),\ f_3(\Omega_{q1q2})$ -scattering amplitude of pi on with clusters and multiple scattering. First part of relation (16), express independent amplitude collision of incident particle with single quarkonium cluster, second part express collision with single second quarkonium cluster and third express double collision with quarkonium clusters. From of this equation, differential cross section for hadron-hadronic scattering determinate as follow:

$$\frac{d\sigma}{d\Omega} = \left| f(\theta,\varphi) \right|^2, \quad (20)$$

with results of equation (20), we can be easily obtained differential cross section of clusters for hadron-hadronic in two quarkonium cluster model.

3. Conclusion

This article focuses on quark clustering targets in hadrons. Elementary quarkonium model treat quadroquark or pentaquark as systems of quarks. These models neglect the internal structure of the quarkonium and effects of the Pauli principle between the quarks in the quarkonium clusters are taken into account by introducing short range repulsion between the clusters. Nowadays cluster formation theory for exotics baryons and mesons is fully proved and satiable among other methods to study particle physic. Clustering target relates to important subject to semi stabilize targets, knowing the method to such end leads us to more accurate and clear cluster type. Choosing cluster model of two quarkonium bound state systems for semi stable hadronof hadronic targets in hadron-hadronic interaction; we will have more stable targets and applying Glouber Theory in high energies. We have accurately calculated scattering amplitude of jet hadron collision with clusters of two quarkonium systems, and result from surveying the oretical determination, clarifies that it is only sum results of (jet hadron,hadronic target), which encompasses all once and twice repeated collision between clusters and jet hadron, while other calculation only illustrate collision of jet hadron with single clusters separately. Results again clarify that in surveying of hadrons collision process with hadronic targets in the high energy approximation, scattering is happening in all quarks and quarkonium diffraction process of the particle under this condition is complete and concentrated and this the best state which different properties of the hadrons and collision can be described and explained.

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