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Bayesian analysis of exponentiated gamma distribution under type II censored samples

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ABSTRACT

The paper is concerned with posterior analysis of exponentiated gamma distribution for type II censored samples. The expressions for Bayes estimators and associated risks have been derived under different priors. The entropy and quadratic loss functions have been assumed for estimation. The posterior predictive distributions have been obtained and corresponding intervals have been constructed. The study aims to find out a suitable estimator of the parameter of the distribution. The findings of the study suggest that the performance of estimators under gamma prior using entropy loss function is the best.

1. Introduction

The exponential distribution has been widely used in time to failure data analysis and is preferred for situations where hazard rate is constant. In case of monotonic hazard rate, a number of distributions have been suggested but Weibull and gamma distributions are mostly used. The gamma distribution has a major constraint that its distribution function and survival function cannot be expressed in nice closed forms which create difficulties for further mathematical manipulations. The distribution function, the survival function or the hazard function for the distribution are often evaluated numerically. This is one of the vital reasons that made the gamma distribution unpopular in comparison to the Weibull distribution. Although Weibull distribution has a nice closed form for hazard and survival function, but it has its own disadvantages. For example, Bain and Engelhardt (1991) have indicated that the maximum likelihood estimators (MLE's) for the parameters of the Weibull distribution may not behave properly over the whole parametric space. Gupta et al. (1998) proposed the use of the exponentiated gamma distribution as an alternative to gamma and Weibull distributions. Shawkyand and Bakoban (2008) discussed the exponentiated gamma distribution as an important model of life time models and derived Bayesian and non-Bayesian estimators of the shape parameter, reliability and failure rate functions in the case of complete and type-II censored samples. The mean square errors of the estimates were computed. Comparisons were made between these estimators using Monte Carlo simulation study. Shawky and Bakoban (2009) addressed the problem of estimating the parameters, reliability and failure rate functions of the finite mixture of two components from exponentiated gamma distributions. The maximum likelihood and Bayes methods of estimation were used for estimation of the said parameter. Shawkyand and Bakoban (2009) considered the order statistics from an exponentiated gamma (EG) distribution. Exact expressions for the single and double moments of order statistics from EG distribution were derived. Based on the moments of order statistics, the best linear unbiased estimators (BLUE's) for the location and scale parameters of EG distribution under Type-II censoring were obtained. Finally, comparisons between the estimators are made based on simulation study. Persson and Ryden (2010) discussed the estimation of T-year return values for significant wave height in a case study and compare point estimates and their uncertainties to the results given by alternative approaches using Gumbel or Generalized Extreme Value distributions. Ghanizadeh et al. (2011) dealt with the estimation of parameters of the Exponentiated Gamma (EG) distribution with presence of k outliers. The maximum likelihood and moment of the estimators were derived. These estimators are compared empirically using Monte Carlo simulation. Khan and Kumar (2011) established the explicit expressions and some recurrence relations for single and product moments of lower generalized order statistics from exponentiated gamma distribution. Singh et al. (2011) proposed Bayes estimators of the parameter of the Exponentiated gamma distribution and associated reliability function under General Entropy loss function for a censored sample. The proposed estimators were compared with the corresponding Bayes estimators obtained under squared error loss function and maximum likelihood estimators through their simulated risks.

Gupta and Kundu (2001) studied some properties of a new family of distributions, namely Exponentiated Exponential distribution. It was observed that many properties of this new family are quite similar to those of a Weibull or a gamma family; therefore this distribution can be used as a possible alternative to a Weibull or a gamma distribution. Raqab and Madi (2009) considered the Bayesian estimation and prediction for the exponentiated Rayleigh model, using informative and non-informative priors. An importance sampling technique was used to estimate the parameters, as well as the reliability function. The Gibbs and Metropolis samplers were used to predict the behavior of future observations from the distribution. Two data sets were used to illustrate our procedures. Raja and Mir (2011) proposed extension of exponentiated Weibull, exponentiated exponential, exponentiated lognormal and exponentiated Gumbel distributions .Two parameter exponentiated Weibull was found to be suitable to fit unimodel, monotone and risk functions unlike Weibull model. Parameter estimation was done by maximum likelihood estimation. Goodness of fit was carried out for two data sets for illustration. Flaih et al. (2012) considered the standard exponentiated inverted Weibull distribution (EIW) that generalizes the standard inverted Weibull distribution (IW), the new distribution has two shape parameters. The moments, median, survival function, hazard function, maximum likelihood estimators, least-squares estimators, fisher information matrix and asymptotic confidence intervals were discussed. A real data set was analyzed and it was observed that the (EIW) distribution can provide a better fitting than (IW) distribution. Zea et al. (2012) introduced and studied the beta exponentiated Pareto distribution. Its density and failure rate functions can have different shapes. It contains as special models several important distributions discussed in the literature, such as the beta-Pareto and exponentiated Pareto distributions. A comprehensive mathematical treatment of the distribution and expressions for the moments generating function and quantile functions and incomplete and L-moments were derived and presented.

The probability density function (pdf) of the exponentiated gamma distribution is:

$$f(x)=\theta x e^{-x}\left\{1-e^{-x}(x+1)\right\}^{\theta-1} \quad ; \quad \theta>0 \tag{1}$$

The cumulative distribution function (CDF) of the distribution is:

$$F(x)=\left\{1-e^{-x}(x+1)\right\}^{\theta} \quad ; \quad \theta>0 \tag{2}$$

2. Material and methods

This section includes the derivation of posterior distributions, Bayes estimates, posterior risks, posterior predictive distributions and posterior predictive intervals.

2.1. Posterior distributions under type II censored samples

Suppose an experiment consists of a fixed number of items (say n) but the experiment is stopped when a predetermined number (say k) are observed to have failed; the remaining (n – k) items are still working. However, the information regarding the items, which are still working, is not of interest. Then these items may be called as censored items and the corresponding procedure will said to be Type II censoring. The likelihood function for (n – k) type II censored sample is:

$$L\left(\theta|\underline{x}\right)\propto\left\{\prod_{i=1}^{k}f\left(x_i\right)\right\}\left\{1-F\left(x_k\right)\right\}^{n-k} \tag{3}$$

After simplifications it becomes:

$$L\left(\theta|\underline{x}\right)\propto\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\theta^k e^{-\theta\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]} \tag{4}$$

The efficiency of Bayesian framework is largely dependent upon the choice of an appropriate prior distribution. The prior information is combined to the current information to update the belief regarding a particular characteristic of the data. The prior information can be of two types; informative and non-informative priors. Though, the choice of a prior depends upon the circumstances of the study but the search for a suitable prior is always of interest. We have utilized informative and non-informative prior for posterior analysis of the exponentiated gamma distribution. The posterior distributions under each prior are presented in the following.

The uniform prior is assumed to be: $p(\theta)\propto 1$; $\theta>0$ (5)

The posterior distribution under the assumption of uniform prior is:

$$p\left(\theta|\underline{x}\right)=\frac{\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\theta^k e^{-\theta\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]}}{\Gamma(k+1)\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+1)}}\ ;\ \theta>0 \tag{6}$$

The Jeffreys prior is derived to be: $p(\theta)\propto\frac{1}{\theta}$; $\theta>0$ (7)

The posterior distribution under the assumption of Jeffreys prior is:

$$p\left(\theta|\underline{x}\right)=\frac{\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\theta^{k-1} e^{-\theta\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]}}{\Gamma(k)\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k)}}\ ;\ \theta>0 \tag{8}$$

The exponential prior is assumed to be:

$$p(\theta)=ae^{-a\theta}\ ;\quad a>0,\ \theta>0 \tag{9}$$

Where a is hyper parameter.

The posterior distribution under the assumption of exponential prior is:

$$p\left(\theta|\underline{x}\right)=\frac{\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\theta^{k}e^{-\theta\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+a\right]}}{\Gamma(k+1)\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+a\right]^{-(k+1)}}\quad ;\ \theta>0 \qquad (10)$$

The gamma prior is assumed to be:

$$p(\theta)=\frac{c^{b}}{\Gamma(b)}\theta^{b-1}e^{-\theta c}\quad ,\ \theta>0,\ b,c>0 \qquad (11)$$

Where b and c are hyper parameters.

The posterior distribution under the assumption of gamma prior is:

$$p\left(\theta|\underline{x}\right)=\frac{\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\theta^{k+b-1}e^{-\theta\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+c\right]}}{\Gamma(k+b)\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+c\right]^{-(k+b)}}\quad ;\ \theta>0 \qquad (12)$$

The chi square prior is assumed to be:

$$p(\theta)=\frac{\theta^{\frac{d}{2}}e^{-\frac{\theta}{2}}}{\Gamma\left(\frac{d}{2}\right)2^{\frac{d}{2}}}\quad ;\qquad \theta>0\ \ ;\ d>0 \qquad (13)$$

Where; d is the hyper parameter.

The posterior distribution under the assumption of chi square prior is:

$$p\left(\theta|\underline{x}\right)=\frac{\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\theta^{k+\frac{d}{2}-1}e^{-\theta\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+\frac{1}{2}\right]}}{\Gamma\left(k+\frac{d}{2}\right)\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+\frac{1}{2}\right]^{-\left(k+\frac{d}{2}\right)}}\quad ;\ \theta>0 \qquad (14)$$

2.2. Bayes estimators and posterior risks

In this section, the Bayes estimator and associated posterior risks have been derived under the assumption of uniform prior using entropy and quadratic loss functions.

Bayes estimators and risks under uniform prior using entropy loss function are:

$$\theta_{ELF}=\frac{k\sum_{i=1}^{n-k}(-1)^{j}\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}\right]^{-(k+1)}}{\sum_{j=0}^{n-k}(-1)^{j}\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}\right]^{-(k)}} \qquad (15)$$

$$\rho\left(\theta_{ELF}\right)=\frac{\sum_{i=1}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{w=0}^{k}\frac{1}{w}-\gamma-\ln\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]\right]}{\Gamma(k+1)\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+1)}}$$
$$-\ln\left[\frac{k\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+1)}}{\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k)}}\right] \tag{16}$$

Bayes estimators and risks under uniform prior using quadratic loss function are:

$$\theta_{QLF}=\frac{(k-1)\sum_{i=1}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k)}}{\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k-1)}} \tag{17}$$

$$\rho\left(\theta_{QLF}\right)=1-\frac{(k-1)\left[\sum_{i=1}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k)}\right]^2}{k\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+1)}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k-1)}} \tag{18}$$

The expressions for estimators and risks under Jeffreys, exponential, gamma and chi square priors can be derived in a similar manner.

2.3. Posterior predictive distributions and intervals

Posterior predictive distribution is defined as: $p\left(y|\underline{x}\right)=\int_0^{\infty}p(\theta|\underline{x})f\left(y;\theta\right)d\theta$ (19)

The posterior predictive distribution under uniform prior is:

$$p\left(\theta|\underline{x}\right)=\frac{(k+1)ye^{-y}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}+\ln\left\{1-e^{-y}\left(y+1\right)\right\}\right]^{-(k+2)}}{\left\{1-e^{-y}\left(y+1\right)\right\}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+1)}} \tag{20}$$

The posterior predictive distributions under other priors can be derived accordingly.

The posterior predictive interval under uniform prior can be obtained by solving the following equations: $\int_0^{L}p\left(y|\underline{x}\right)dy=\frac{\alpha}{2}$, $\int_U^{\infty}p\left(y|\underline{x}\right)dy=\frac{\alpha}{2}$; where $1-\alpha$ is the confidence coefficient.

After simplifications the above equations become:

$$\frac{(k+1)ye^{-y}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}+\ln\left\{1-e^{-L}\left(L+1\right)\right\}\right]^{-(k+2)}-\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+2)}\right]}{\left\{1-e^{-y}\left(y+1\right)\right\}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}\left(x_i+1\right)\right\}^{-1}+j\ln\left\{1-e^{-x_k}\left(x_k+1\right)\right\}^{-1}\right]^{-(k+1)}}=\frac{\alpha}{2} \tag{21}$$

$$\frac{(k+1)ye^{-y}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}+\ln\left\{1-e^{-U}(U+1)\right\}\right]^{-(k+2)}-\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}\right]^{-(k+2)}\right]}{\left\{1-e^{-y}(y+1)\right\}\sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left[\sum_{i=1}^{k}\ln\left\{1-e^{-x_i}(x_i+1)\right\}^{-1}+j\ln\left\{1-e^{-x_k}(x_k+1)\right\}^{-1}\right]^{-(k+1)}}=1-\frac{\alpha}{2} \quad (22)$$

The posterior predictive intervals under remaining priors can be constructed accordingly.

3. Results and discussions

The simulation study has been carried out under different samples sizes under 1000 replications. The results have been obtained for $\theta \in (2,4,6,8,10)$. The purpose of the simulation study is to compare the performance of the estimates in terms of magnitudes of associated posterior risks.

Table 1
Bayes estimates and risks under different priors for θ = 2 using 10% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	2.17501	2.19732	2.13151	2.15337	2.05469	2.07577	2.03032	2.05114	2.16451	2.18671
	0.01913	0.00948	0.01952	0.00968	0.01913	0.00948	0.00010	0.00005	0.03001	0.01488
100	2.09845	2.11997	2.07746	2.09877	2.04080	2.06173	2.02871	2.04951	2.09490	2.11638
	0.00959	0.00476	0.00969	0.00480	0.00959	0.00476	0.00009	0.00004	0.02701	0.01339
150	2.07471	2.09598	2.06087	2.08201	2.03678	2.05767	2.02874	2.04954	2.07268	2.09394
	0.00641	0.00318	0.00639	0.00317	0.00641	0.00318	0.00007	0.00004	0.02161	0.01071
200	2.06648	2.08768	2.05271	2.07376	2.02871	2.04952	2.02070	2.04142	2.06447	2.08564
	0.00427	0.00212	0.00425	0.00211	0.00427	0.00212	0.00006	0.00003	0.01728	0.00857
300	2.05829	2.07940	2.04457	2.06554	2.02067	2.04139	2.01269	2.03333	2.05629	2.07737
	0.00284	0.00141	0.00283	0.00140	0.00284	0.00141	0.00005	0.00002	0.01383	0.00686

Table 2
Bayes estimates and risks under different priors for θ = 4 using 10% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	4.32891	4.37331	4.24233	4.28584	4.08944	4.13138	4.04092	4.08236	4.30801	4.35219
	0.03221	0.01597	0.03287	0.01630	0.03221	0.01597	0.00017	0.00008	0.05054	0.02506
100	4.17653	4.21936	4.13476	4.17716	4.06179	4.10344	4.03772	4.07913	4.16945	4.21221
	0.01616	0.00801	0.01632	0.00809	0.01616	0.00801	0.00015	0.00007	0.04548	0.02255
150	4.12927	4.17161	4.10174	4.14380	4.05379	4.09536	4.03778	4.07919	4.12524	4.16755
	0.01080	0.00535	0.01076	0.00533	0.01080	0.00535	0.00012	0.00006	0.03639	0.01804
200	4.11290	4.15508	4.08548	4.12738	4.03772	4.07913	4.02178	4.06302	4.10889	4.15103
	0.00719	0.00356	0.00716	0.00355	0.00719	0.00356	0.00010	0.00005	0.02911	0.01443
300	4.09660	4.13862	4.06929	4.11103	4.02172	4.06297	4.00584	4.04692	4.09261	4.13458
	0.00478	0.00237	0.00477	0.00236	0.00478	0.00237	0.00008	0.00004	0.02329	0.01155

Table 3
Bayes estimates and risks under different priors for θ = 6 using 10% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	6.46523	6.53153	6.33593	6.40090	6.10758	6.17021	6.03511	6.09700	6.43402	6.50000
	0.04027	0.01996	0.04109	0.02037	0.04027	0.01996	0.00021	0.00010	0.06317	0.03132
100	6.23764	6.30161	6.17526	6.23859	6.06628	6.12849	6.03034	6.09218	6.22708	6.29094
	0.02020	0.01001	0.02040	0.01012	0.02020	0.01001	0.00019	0.00009	0.05686	0.02819
150	6.16706	6.23031	6.12595	6.18877	6.05434	6.11642	6.03042	6.09227	6.16105	6.22423
	0.01349	0.00669	0.01345	0.00667	0.01349	0.00669	0.00015	0.00007	0.04548	0.02255
200	6.14262	6.20562	6.10167	6.16425	6.03034	6.09218	6.00653	6.06812	6.13663	6.19956
	0.00898	0.00445	0.00895	0.00444	0.00898	0.00445	0.00012	0.00006	0.03639	0.01804
300	6.11828	6.18102	6.07749	6.13982	6.00644	6.06804	5.98272	6.04408	6.11231	6.17499
	0.00598	0.00296	0.00596	0.00295	0.00598	0.00296	0.00010	0.00005	0.02911	0.01443

Table 4
Bayes estimates and risks under different priors for θ = 8 using 10% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	8.60544	8.69369	8.43334	8.51982	8.12940	8.21277	8.03294	8.11532	8.56390	8.65172
	0.05235	0.02595	0.05341	0.02648	0.05235	0.02595	0.00027	0.00013	0.08212	0.04072
100	8.30251	8.38766	8.21949	8.30378	8.07442	8.15723	8.02659	8.10890	8.28846	8.37346
	0.02626	0.01302	0.02652	0.01315	0.02626	0.01302	0.00024	0.00012	0.07391	0.03664
150	8.20857	8.29275	8.15385	8.23747	8.05853	8.14117	8.02670	8.10902	8.20057	8.28467
	0.01754	0.00870	0.01749	0.00867	0.01754	0.00870	0.00019	0.00010	0.05913	0.02932
200	8.17604	8.25989	8.12154	8.20482	8.02659	8.10891	7.99489	8.07688	8.16807	8.25183
	0.01168	0.00579	0.01164	0.00577	0.01168	0.00579	0.00015	0.00008	0.04730	0.02345
300	8.14364	8.22715	8.08935	8.17231	7.99478	8.07677	7.96321	8.04487	8.13570	8.21913
	0.00777	0.00385	0.00775	0.00384	0.00777	0.00385	0.00012	0.00006	0.03784	0.01876

Table 5
Bayes estimates and risks under different priors for θ = 10 using 10% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	10.74566	10.85586	10.53074	10.63874	10.15122	10.25532	10.03077	10.13364	10.69378	10.80345
	0.06443	0.03194	0.06574	0.03259	0.06443	0.03194	0.00033	0.00016	0.10108	0.05011
100	10.36739	10.47371	10.26371	10.36897	10.08257	10.18597	10.02284	10.12563	10.34984	10.45598
	0.03232	0.01602	0.03264	0.01618	0.03232	0.01602	0.00030	0.00015	0.09097	0.04510
150	10.25008	10.35520	10.18175	10.28616	10.06272	10.16592	10.02298	10.12577	10.24009	10.34510
	0.02159	0.01070	0.02152	0.01067	0.02159	0.01070	0.00024	0.00012	0.07278	0.03608
200	10.20946	10.31416	10.14140	10.24540	10.02284	10.12563	9.98326	10.08564	10.19951	10.30410
	0.01437	0.00712	0.01432	0.00710	0.01437	0.00712	0.00019	0.00009	0.05822	0.02886
300	10.16900	10.27329	10.10121	10.20480	9.98312	10.08550	9.94370	10.04567	10.15909	10.26327
	0.00956	0.00474	0.00953	0.00473	0.00956	0.00474	0.00015	0.00008	0.04658	0.02309

Table 6
Bayes estimates and risks under different priors for θ = 2 using 20% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	2.19644	2.21897	2.15251	2.17459	2.07494	2.09622	2.05032	2.07134	2.18584	2.20826
	0.01993	0.00988	0.02034	0.01008	0.01993	0.00988	0.00010	0.00005	0.03127	0.01550
100	2.11912	2.14086	2.09793	2.11945	2.06091	2.08204	2.04870	2.06971	2.11554	2.13723
	0.01000	0.00496	0.01010	0.00501	0.01000	0.00496	0.00009	0.00005	0.02814	0.01395
150	2.09515	2.11663	2.08118	2.10252	2.05685	2.07794	2.04873	2.06974	2.09310	2.11457
	0.00668	0.00331	0.00666	0.00330	0.00668	0.00331	0.00007	0.00004	0.02251	0.01116
200	2.08684	2.10824	2.07293	2.09419	2.04870	2.06971	2.04061	2.06153	2.08481	2.10619
	0.00445	0.00220	0.00443	0.00220	0.00445	0.00220	0.00006	0.00003	0.01801	0.00893
300	2.07857	2.09989	2.06472	2.08589	2.04058	2.06151	2.03252	2.05336	2.07655	2.09784
	0.00296	0.00147	0.00295	0.00146	0.00296	0.00147	0.00005	0.00002	0.01441	0.00714

Table 7
Bayes estimates and risks under different priors for θ = 4 using 20% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	4.37135	4.41618	4.28393	4.32786	4.12953	4.17188	4.08054	4.12238	4.35025	4.39486
	0.03423	0.01697	0.03492	0.01731	0.03423	0.01697	0.00018	0.00009	0.05370	0.02662
100	4.21747	4.26072	4.17530	4.21811	4.10161	4.14367	4.07731	4.11912	4.21033	4.25351
	0.01717	0.00851	0.01734	0.00860	0.01717	0.00851	0.00016	0.00008	0.04833	0.02396
150	4.16975	4.21251	4.14195	4.18443	4.09353	4.13551	4.07737	4.11918	4.16569	4.20840
	0.01147	0.00569	0.01143	0.00567	0.01147	0.00569	0.00013	0.00006	0.03866	0.01917
200	4.15323	4.19582	4.12554	4.16785	4.07731	4.11912	4.06121	4.10286	4.14918	4.19173
	0.00763	0.00379	0.00761	0.00377	0.00763	0.00379	0.00010	0.00005	0.03093	0.01533
300	4.13677	4.17919	4.10919	4.15133	4.06115	4.10280	4.04511	4.08660	4.13273	4.17511
	0.00508	0.00252	0.00506	0.00251	0.00508	0.00252	0.00008	0.00004	0.02474	0.01227

Table 8
Bayes estimates and risks under different priors for θ = 6 using 20% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	6.59899	6.66667	6.46701	6.53333	6.23394	6.29787	6.15998	6.22315	6.56714	6.63448
	0.04228	0.02096	0.04314	0.02139	0.04228	0.02096	0.00022	0.00011	0.06633	0.03289
100	6.36670	6.43199	6.30303	6.36767	6.19179	6.25528	6.15511	6.21823	6.35592	6.42110
	0.02121	0.01052	0.02142	0.01062	0.02121	0.01052	0.00020	0.00010	0.05970	0.02960
150	6.29466	6.35921	6.25269	6.31681	6.17960	6.24297	6.15519	6.21831	6.28852	6.35301
	0.01417	0.00703	0.01412	0.00700	0.01417	0.00703	0.00016	0.00008	0.04776	0.02368
200	6.26971	6.33401	6.22791	6.29178	6.15511	6.21823	6.13080	6.19367	6.26360	6.32783
	0.00943	0.00468	0.00940	0.00466	0.00943	0.00468	0.00012	0.00006	0.03821	0.01894
300	6.24486	6.30891	6.20323	6.26685	6.13071	6.19359	6.10650	6.16913	6.23877	6.30275
	0.00628	0.00311	0.00626	0.00310	0.00628	0.00311	0.00010	0.00005	0.03057	0.01515

Table 9
Bayes estimates and risks under different priors for θ = 8 using 20% censored samples.

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	8.73921	8.82883	8.56442	8.65225	8.25576	8.34043	8.15781	8.24147	8.69702	8.78621
	0.05638	0.02795	0.05752	0.02852	0.05638	0.02795	0.00029	0.00014	0.08844	0.04385
100	8.43157	8.51804	8.34725	8.43286	8.19993	8.28402	8.15136	8.23495	8.41730	8.50362
	0.02828	0.01402	0.02856	0.01416	0.02828	0.01402	0.00026	0.00013	0.07960	0.03946
150	8.33617	8.42166	8.28059	8.36551	8.18379	8.26772	8.15147	8.23506	8.32804	8.41344
	0.01889	0.00937	0.01883	0.00934	0.01889	0.00937	0.00021	0.00010	0.06368	0.03157
200	8.30313	8.38828	8.24778	8.33236	8.15136	8.23495	8.11917	8.20243	8.29503	8.38010
	0.01257	0.00623	0.01253	0.00621	0.01257	0.00623	0.00017	0.00008	0.05094	0.02526
300	8.27023	8.35504	8.21509	8.29934	8.11905	8.20232	8.08699	8.16992	8.26216	8.34689
	0.00837	0.00415	0.00834	0.00414	0.00837	0.00415	0.00013	0.00007	0.04075	0.02021

Table 10
Bayes estimates and risks under different priors for θ= 10 using 20% censored samples

n	Uniform		Jeffreys		Exponential		Gamma		Chi Square	
	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF	QLF	ELF
50	10.83483	10.94595	10.61814	10.72703	10.23546	10.34043	10.11401	10.21774	10.78253	10.89310
	0.06846	0.03394	0.06985	0.03463	0.06846	0.03394	0.00035	0.00017	0.10739	0.05324
100	10.45342	10.56063	10.34889	10.45502	10.16624	10.27050	10.10602	10.20966	10.43573	10.54275
	0.03434	0.01702	0.03468	0.01720	0.03434	0.01702	0.00032	0.00016	0.09665	0.04792
150	10.33515	10.44113	10.26625	10.37153	10.14623	10.25028	10.10616	10.20980	10.32507	10.43095
	0.02294	0.01137	0.02287	0.01134	0.02294	0.01137	0.00025	0.00013	0.07732	0.03834
200	10.29419	10.39976	10.22556	10.33042	10.10602	10.20966	10.06611	10.16934	10.28415	10.38961
	0.01527	0.00757	0.01522	0.00755	0.01527	0.00757	0.00020	0.00010	0.06186	0.03067
300	10.25339	10.35854	10.18504	10.28948	10.06597	10.16920	10.02622	10.12904	10.24339	10.34844
	0.01016	0.00504	0.01013	0.00502	0.01016	0.00504	0.00016	0.00008	0.04949	0.02453

It is evident from the above analysis that the increased samples size imposes a positive impact on the behavior of the estimators. On the other hand, the increasing true parametric values negatively affected the performance of the estimators. The amount of over-estimation has been observed under each prior, sample size, loss function and censoring rate. In case of non-informative priors, the estimates under Jeffreys priors are having the larger efficiencies as compared to those under uniform prior. While for informative priors, the performance of the estimates under gamma prior is significantly better than others. In comparison of informative and non-informative priors it is assessed that the estimates under the informative priors are simply better than those under non-informative priors. Similarly, the estimates under entropy loss function are associated with the lesser amounts of posterior risks. The increased censoring rate results in inflation of the corresponding magnitudes of risks; whereas, the convergence rate becomes slower due to increase in censoring rate.

4. Conclusion

The study was conducted to find out an appropriate Bayes estimator for the parameter of exponentiated gamma distribution. The study proposed that in order estimate the said parameter the use of gamma prior under entropy loss function can be preferred.

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